

## Equilibrium of Price Discrimination for Churn-in: Considering Consumer Re-switching

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This paper analyzes price discrimination where a firm price-discriminates between its own and rival's customers. In such situations, consumers can seek more benefits through repetitive re-switching between the firm and its rival. However, only a few previous studies have addressed such re-switching of consumers and unrealistic restrictive assumptions have been made. The model in this paper deals with repetitive re-switching with the assumption of arbitrary number of stages and multi-period benefit maximization of consumers. Our model derives the range of initial market share which provides tenable equilibrium price. Such consideration of multi-stage equilibrium helps managers to make more meaningful market expectations. Our analysis also reveals that the condition of initial market share for tenable equilibrium includes plausible results before the introduction of price discrimination. The consumers' re-switching frustrates the firms' efforts to attract its rival's customers in the next period. Moreover, considering re-switching, the discount offered by a firm for switchers not only attracts its rival's customers but also sacrifices its own customers. Therefore, re-switching by consumers alleviates price competition between the two firms.

Key words: Price discrimination, Product differentiation, Linear-city, Switching

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### I. Introduction

With the advancement of information technology, the importance of market segmentation and price discrimination is growing for achieving market success. Shaffer and Zhang (1995) notes that "with the advent of panel data on household purchase behavior and the development of statistical procedures to utilize this data, firms can now target coupons to

selected households with considerable accuracy and cost effectiveness." Furthermore, Taylor (2003) also adds that "a retailer might use information it has obtained from its previous dealings with a customer to offer that customer a special deal (or, sometimes, a bad deal)." In particular, third-degree price discrimination is widely observed in markets because a firm can easily distinguish between its own (who have used its service) and its rival's customers (who have used its rival's service).<sup>1)</sup> Examples

of this phenomenon may be readily found especially in subscription markets such as cable TVs, mobile communications, internet services, newspapers, and magazines. Since firms in subscription markets provide a flow of goods or services directly to their customers, the transactions are not anonymous. Every firm recognizes whether or not a particular consumer is one of its current subscribers and may price-discriminate on the basis of this knowledge (Taylor, 2003).

This type of third-degree price discrimination is called behavior-based price discrimination, because different groups of consumers are recognized on the basis of their purchase behavior. The previous studies that are available with respect to this issue may be classified into two categories. The first includes the studies of a few researchers including Chen (1997) and Taylor (2003) which assumed homogeneous goods and switching costs. The second includes the studies that applied the linear-city model, which assumes heterogeneous goods.

Previous studies that conducted on the linear-city model have derived various results depending on the structure of their models. Thisse and Vives (1988), Shaffer and Zhang (1995), and Bester and Petrakis (1996) found that in an oligopoly, the profits made by firms when they price-discriminate are always lower as compared to those made under the uniform pricing. This intense competition is called a prisoner's dilemma because firms select price

discrimination in order to improve their profits; however, their equilibrium payoffs are worse than the uniform pricing regime. This result is interesting because it is contrary to what has been reasoned in a monopoly.

Other researchers have found that prisoner's dilemma does not occur under certain circumstances. Kim et al. (2001) and Singh et al. (2008) derived such results by assuming only the reward programs, which provide an incentive for protecting its own customers. Kim et al. (2001) reasoned that the reward programs enable firms to reach collusion. Singh et al. (2008) indicated that an equilibrium can be reached when one or both firms adopt reward programs. Shaffer and Zhang (2000, 2002) derived the similar result on the basis of asymmetric brand loyalties of consumers.

Fudenberg and Tirole (2000) focused on the impact of long-term contract and consumers' expectations on future price discrimination. They indicated that price discrimination in the second period may increase the uniform price in the first period and a short-term contract is more attractive for firms even though a long-term contract may reduce inefficient switching. Pazgal and Soberman (2008) extended the work of Fudenberg and Tirole (2000) by additionally assuming that firms can add benefits to their offers for past customers in the second period.

Although many studies have dealt with price discrimination for churn-in as mentioned

above, there still remain important points that have not received sufficient research attention. In subscription markets such as mobile communications, repetitive re-switching is common. Our customers acquired in this period may switch to the other firm in the next period. Such repetitive re-switching means that consumers acquired by price discrimination are not easy to maintain for a long time. Therefore, repetitive re-switching can affect the incentive of firms for the discrimination. Song (2009, 2010), to our best knowledge, was the first researcher who considered the case where consumers repetitively re-switch over multiple periods. However, Song (2009, 2010) had some important limitations. Song (2009) considered only symmetric firms with respect to brand quality and cost and failed to include a discount factor even though multiple periods were considered. While Song (2010) proposed a more elaborate model by introducing asymmetry between firms and a discount factor, his model dealt with only the case where consumers maximize the benefit of just one period and initial market share is given the same as the equilibrium market share in uniform pricing. Rational consumers are expected to maximize the total benefit considering the future as well as the present. From such a traditional perspective, Song (2010) dealt with only a specific situation where consumers are unable to predict future. Moreover, if firms anticipate the introduction of

price-discrimination and the impact of initial market share, they can adjust their uniform price from traditional uniform price equilibrium. Then, we should analyze the robustness of equilibrium considering re-switching with respect to initial market share.

With this background, the model in this paper deals with repetitive re-switching with the assumption of arbitrary number of stages and multi-period benefit maximization of consumers. Moreover, we derive the range of initial market share which can give equilibrium under consumers' repetitively re-switching. As a result, this paper develops more generalized equilibrium. In the equilibrium, consideration of re-switching by consumers reduces a firm's incentive to discriminate price. The first reason is that the consumers who were attracted on the basis of price-discount revert to the rival during the subsequent period, when consumers are permitted to re-switch. In other words, re-switching by consumers during the subsequent period negates the efforts made by firms for attracting their rival's consumers during the subsequent period. The second reason is that although a firm's price discount for switchers attracts its rival's customers, it sacrifices its own customers.

The remaining paper is organized in the following manner. Section 2 presents an analytic framework. Section 3 analyzes the equilibrium prices and profits. Finally, Section 4 summarizes and concludes the study.

## II. Model

A linear-city duopoly model is adopted in this paper. Firms  $A$  and  $B$  produce one type of products each. The marginal costs for both of the firms are assumed equal and normalized to zero. The fixed cost is not considered. All consumers have unit demands in every period. The marketplace is expressed as the line segment  $[0,1]$ , where it is assumed that the two firms (or products) are located at either end, i.e.,  $A$  at 0 and  $B$  at 1. Consumers are considered continuously and uniformly situated along the line.

Consumers are required to bear the characteristic transportation cost if their position in the marketplace is distant from that of the product that they intend to purchase. Considering transportation cost, consumers located at  $x \in [0,1]$  receive  $U_A - P_A - tx$  as their net benefit (or utility) if they purchase  $A$ , or  $U_A + u - P_B - t(1-x)$  if they purchase  $B$ .  $P_i$  ( $i = A, B$ ) and  $t$  denote the price of the product and transportation cost for unit distance, respectively.  $U_A$  denotes the benefit (or base value) of Product  $A$ .  $u$  ( $t \geq u \geq 0$ ) implies relatively higher brand quality of  $B$ , and  $t \geq u$  implies that there exist some consumers who prefer  $A$  over  $B$ , when prices of both the firms are equal.  $U_A$  is sufficiently large for all the consumers to purchase either  $A$  or  $B$  at any given price. None of these values or

consumer preferences, which is denoted by  $x$ , change overtime.

The proposed model assumes an arbitrary number of stages, and one stage is composed of two periods. At the beginning of Period 1, firms  $A$  and  $B$  determine their prices non-cooperatively and simultaneously. The prices cannot be changed in Period 2. In case of uniform pricing, firms determine only their regular price,  $P_A$  and  $P_B$ . However, in the case of price discrimination, they also determine  $P_a$  and  $P_b$  ( $P_A \geq P_a$  and  $P_B \geq P_b$ ), which are applied to switchers. A firm's switchers are those consumers who had selected its rival's product during the previous period, and select the firm's product in the current period. In this case, the regular prices are intended for the stayers, who continue to purchase the product of the same firm that they had selected in the previous period.

The assumption that the prices cannot be changed in Period 2 is somewhat restrictive; however, it reflects reality. First, firms cannot change prices frequently owing to reputation, agreements, or regulations. Under such circumstances, consumers select their suppliers several times at the same prices. Second, as will be indicated in the later analysis, some consumers change their original choices during the first period and revert to the original choices in the second period. This creates different incentives between the two periods and prices change considerably in each period

if the restriction is not assumed. However, the equilibrium in which prices change repetitively at each period is hard to be considered as a market expectation. The previous studies that did not consider the repetitive switching of consumers were not required to consider this aspect. On the other hand, in the real world, consumers do not purchase products simultaneously, and the time period between repurchases varies. Subsequently, firms must establish their pricing policies considering both switching and re-switching (returning to original choices) by consumers. The restriction that firms cannot change their prices in period 2 implies that firms must offer the prices which will be applied to both switching and re-switching, and that consumers who maximize their benefits over multi-periods believe that the prices will remain constant.

On the other hand, in order to adopt price discrimination between its own and its rival's consumers, choices of consumers must be defined prior to Stage 1. Consequently, it is assumed that consumers who are in  $[0, x_0]$  chose  $A$  and those who are in  $[x_0, 1]$  chose  $B$  prior to Stage 1. In other words, a series of stages prior to Stage 1, which introduce price discrimination, are assumed to be omitted from the model and the result is assumed exogenously for simplicity. Hereafter, we call  $x_0$  'base-line market share.' After the equilibrium prices and the conditions for equilibrium are induced, the plausibility of

the exogenously assumed variable  $x_0$  will be evaluated.

The same discount factor  $\delta$  ( $0 \leq \delta \leq 1$ ) is assumed for both firms and consumers. The discount factor has dual meanings. First, in the normal sense, it means the extent to which firms and consumers discount future value. Second, with regard to the proposed model, it means whether or not re-switching by consumers is considered. If  $\delta = 0$ , since only one period is considered in pricing, consumers' re-switching is not considered. If  $\delta$  increases, firms increasingly consider re-switching by consumers.

### III. Analyses

This section analyzes the equilibrium prices and profits. Although the main purpose of this paper is to analyze the characteristics of price discrimination, we will start from uniform pricing and the results will be used for a comparison with those of price discrimination.

#### 3.1 Uniform Pricing

Under a uniform price regime, every period is independent because previous choices of consumers do not affect the choices in the current period. Therefore, an equilibrium that considers only one period is the same as the

one that considers multi-periods. Assuming the net benefits of consumers,  $x_0$  can be defined as the marginal consumer and  $x_0 = (P_B - P_A + t - u) / (2t)$ .

Consumers in  $[0, x_0]$  purchase  $A$ , while consumers in  $[x_0, 1]$  purchase  $B$ . Based on consumers' choices, the profits of  $A$  and  $B$  are represented as  $\Pi_A = P_A(P_B - P_A + t - u) / (2t)$  and  $\Pi_B = P_B(1 - (P_B - P_A + t - u) / (2t))$ , respectively.

Now, we can derive the equilibrium prices by solving the two simultaneous equations obtained from the first-order conditions ( $\partial \Pi_A / \partial P_A = 0, \partial \Pi_B / \partial P_B = 0$ ). The second-order conditions are satisfied because the profit functions are strictly concave in their own prices. The equilibrium prices and choices of consumers are represented in the following manner:

$$P_A^U = t - u/3, P_B^U = t + u/3, \\ \text{and } x_0^U = 1/2 - u/(6t).$$

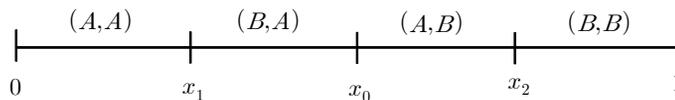
### 3.2 Price Discrimination

When price discrimination is adopted, firms determine  $P_A, P_a, P_B,$  and  $P_b$  ( $P_A \geq P_a$  and  $P_B \geq P_b$ ). In order to induce the equilibrium, initially an arbitrary stage in the game is

independently considered. It is temporarily assumed that the consumers in  $[0, x_0]$  and  $[x_0, 1]$  had selected  $A$  and  $B$ , respectively at the end of the previous stage. And it is temporarily assumed that when firms offer their prices, consumers' choices are indicated in <Figure 1>. In this figure, the first and the second items in the parenthesis refer to the consumer choices in Periods 1 and 2, respectively. Letting  $U_{(i,j)}$  ( $i = A, B$  and  $j = A, B$ ) denote the net benefit from  $(i, j)$ ,  $x_1$  denote the marginal consumer with  $U_{(A,A)}^{Consumers \in [0, x_0]} = U_{(B,A)}^{Consumers \in [0, x_0]}$ , and  $x_2$  denote the marginal consumer with  $U_{(B,B)}^{Consumers \in [x_0, 1]} = U_{(A,B)}^{Consumers \in [x_0, 1]}$ , the following equations are obtained:

$$x_1 = (\delta P_a + P_b - (1 + \delta)P_A + t - u) / (2t) \text{ and} \\ x_2 = ((1 + \delta)P_B - P_a - \delta P_b + t - u) / (2t). \quad (1)$$

The choices of consumers in <Figure 1> bear an important meaning. If a few consumers in  $[0, x_0]$  select  $(B, B)$  or  $(A, B)$ , then the consumers' choices at the beginning of the subsequent stage differ from their choices at the beginning of the current stage. It is only when the choices of consumers are similar to those indicated in <Figure 1> that every stage in the game becomes independent and identical.



<Figure 1> Choices of consumers

Therefore, only when the one-stage equilibrium is derived where the equilibrium choices of consumers are similar to those indicated in <Figure 1> with a specific  $x_0$ , the equilibrium prices will be the same in every stage. This type of equilibrium is meaningful as an expectation of the market and this paper deals with only the equilibrium.<sup>2)</sup>

When the consumers' choice is similar to that indicated in <Figure 1>, the total profits of  $A$  and  $B$  over the two periods are represented as

$$\begin{aligned} \Pi_A &= (1+\delta)P_A x_1 + P_a((x_2 - x_0) + \delta(x_0 - x_1)) \\ \text{and } \Pi_B &= (1+\delta)P_B(1-x_2) \\ &\quad + P_b((x_0 - x_1) + \delta(x_2 - x_0)), \text{ respectively.} \end{aligned}$$

Based on the first-order conditions, the equilibrium prices and consumers choices are represented in the following manner:

$$\begin{aligned} P_A^* &= \frac{(1+5\delta+2(1-3\delta+2\delta^2)x_0)t}{3(1+\delta)} - \frac{u}{3}, \\ P_a^* &= \frac{(3-4(1-\delta)x_0)t}{3} - \frac{u}{3}, \\ P_B^* &= \frac{(3-\delta+4\delta^2-2(1-3\delta+2\delta^2)x_0)t}{3(1+\delta)} + \frac{u}{3}, \\ P_b^* &= \frac{(4\delta-1+4(1-\delta)x_0)t}{3} + \frac{u}{3}, \\ x_1^* &= \frac{t(1+2(1-\delta)x_0+2\delta)-u}{6t}, \\ x_2^* &= \frac{t(3+2(1-\delta)x_0)-u}{6t}. \end{aligned} \quad 3)$$

When  $\delta = 1$ , the prices are represented as

$$P_A^{\delta=1} = P_a^{\delta=1} = t - u/3, \quad P_B^{\delta=1} = P_b^{\delta=1} = t + u/3,$$

and price discrimination does not occur. In this case,  $x_0$  at the end of the stage is  $1/2 - u/(6t)$  regardless of  $x_0$  at the beginning of the stage, and  $x_1^* = x_2^* = x_0$ . All consumers select  $(A,A)$  or  $(B,B)$ . These equilibrium prices and resulting  $x_0$  at the end of the stage correspond to the unique uniform price equilibrium ( $P_A^U$ ,  $P_B^U$ , and  $x_0^U$ ). Therefore, regardless of  $x_0$  prior to Stage 1, the equilibrium prices correspond to those under the uniform price regime if  $\delta = 1$ .

When  $\delta < 1$ ,  $P_i^*$ s can be the equilibrium only for a certain range of  $x_0$  which induces consumers' choices in equilibrium similar to that in <Figure 1>. The proof of equilibrium conditions are shown in the Appendix.

**Proposition 1.** There exists an equilibrium where consumers' choices at the end of every stage are the same, if discount factor ( $\delta$ ) is 1, or if base-line market share ( $x_0$ ) at the beginning of initial stage satisfies

$$\begin{aligned} \frac{(1+\delta+4\delta^2)t - (1+\delta)u}{4(t+2\delta^2t)} &\leq x_0 \\ &\leq \frac{(3-\delta+4\delta^2)t - (1+\delta)u}{4(t+2\delta^2t)} \text{ and} \end{aligned}$$

$$1/3 \leq x_0 \leq 2/3.$$

The range in the condition includes equilibrium market share in uniform price regime

2) This analysis does not prove that there is no equilibrium other than this type of equilibrium.

3) The second-order conditions are satisfied because the profit functions are strictly concave in their own prices. The calculations for the equations here and below are somewhat complex and 'Mathematica 4.1' is used.

if and only if the difference in brand quality ( $u$ ) satisfies  $u \leq 3t/(1+4\delta)$ . The prices in the equilibrium are presented in the following manner:

$$P_A^* = \frac{(1+5\delta+2(1-3\delta+2\delta^2)x_0)t}{3(1+\delta)} - \frac{u}{3},$$

$$P_a^* = \frac{(3-4(1-\delta)x_0)t}{3} - \frac{u}{3},$$

$$P_B^* = \frac{(3-\delta+4\delta^2-2(1-3\delta+2\delta^2)x_0)t}{3(1+\delta)} + \frac{u}{3},$$

$$P_b^* = \frac{(4\delta-1+4(1-\delta)x_0)t}{3} + \frac{u}{3}.$$

Proof. See the Appendix 1. Q.E.D.

Proposition 1 indicates the conditions that the choices of consumers at every stage are similar to those in <Figure 1>, and if the conditions are satisfied, the equilibrium prices in Proposition 1 are maintained uniformly in every stage. The conditions in Proposition 1 include the possible results of the omitted stages prior to adopting price discrimination ( $x_0$  at the beginning of Stage 1). If the conditions of  $x_0$  do not include the possible results of the omitted stages, the equilibrium in Proposition 1 is meaningless. Primarily, the result of uniform pricing, i.e.,  $x_0^U$  can be assumed to be the plausible  $x_0$ . However, when firms expect the introduction of price discrimination, one may believe that they will be rather concerned about  $x_0$  because  $x_0$  affects future market share ( $\partial x_1^*/\partial x_0 \geq 0$ ,  $\partial x_2^*/\partial x_0 \geq 0$ , and  $x_0$  influences the market share in itself) in a manner that is different from the uniform

pricing, in which market share is not affected by the previous market share. Therefore,  $x_0$  can be slightly changed from  $x_0^U$ , and if the conditions of  $x_0$  in Proposition 1 is a range including  $x_0^U$ , we can say that the equilibrium in Proposition 1 can be market expectation. This occurs only when the difference in brand quality ( $u$ ) is relatively low ( $u \leq 3t/(1+4\delta)$ ).  $u \leq 3t/(1+4\delta)$  is satisfied when  $\delta \leq 1/2$ , and if  $\delta > 1/2$ ,  $u$  has a additional restriction for Proposition 1 to be meaningful.

**Proposition 2.** If  $\delta=1$ , the equilibrium prices correspond to those under the uniform price regime. Else, the equilibrium prices are lower than those under uniform price regime. All prices except the regular price of  $B$  increases with an increase in  $\delta$ . Furthermore, profits increase with an increase in  $\delta$ .

Proof. See the Appendix 2. Q.E.D.

An increase in  $\delta$  generally raises the present value because the future values become larger. However, this model also implies that firms and consumers increasingly consider repetitive re-switching. Intuitively, consumer re-switching during Period 2 makes the efforts to attract its rival's consumers in Period 1 less attractive because the switchers in Period 1 revert to their original suppliers during Period 2 as indicated by the choices of consumers in <Figure 1>. In other words, a firm's endeavor to attract its rival's customers during Period

1 proves to be useless in Period 2. Therefore, the price war between the two firms decreases when they focus more on future profit. Furthermore, in Equations (1), which define  $x_1$ ,  $P_a$  is positively included in  $x_1$ . Therefore, if  $A$  fixes a lower  $P_a$  in order to attract  $B$ 's consumers, it has to sacrifice its own stayers ( $dx_1/dP_a > 0$ ). Similarly,  $P_b$  possesses the same characteristics as  $P_a$ . Considering multi-periods, a few consumers select less preferable suppliers in Period 1 in order to take advantage of the additional benefits offered in Period 2 (discounted price for switcher). For example, let us consider consumers with  $x_1 + \epsilon$  ( $\epsilon \rightarrow 0$ ). They prefer  $A$  over  $B$  if they consider only a single period. However, they earn much more by selecting  $(B, A)$  as compared to  $(A, A)$  in Period 2 because  $P_a < P_A$ . In other words, they select  $B$  in Period 1 in order to earn more during Period 2 despite suffering a loss in Period 1. Therefore, we can conclude that lowering  $P_a$  or  $P_b$  has a 'self-squeezing effect.' Since this effect makes poaching less attractive for firms and increases with an increase in  $\delta$  ( $dx_1/dP_a = \delta/(2t)$  and  $dx_2/dP_b = -\delta/(2t)$ ), a higher  $\delta$  decreases the firms' incentive to its poach rival's consumers through price discrimination.

#### IV. Concluding Remarks

In subscription markets such as mobile

communications, repetitive re-switching is common. Consumers are willing to decrease their benefits in the current period by switching, because they want to get a discount in the next period by re-switching. Therefore, firms should consider the consumers' switching over a number of periods.

In this paper, a behavior-based price discrimination, where a firm price-discriminates between its own and rival's customers, has been analyzed using a linear-city model. This type of price discrimination can be specifically observed in subscription markets such as cable TVs, mobile communications, internet services, newspapers, and magazines. In these markets, numerous consumers seek higher benefit through repetitive re-switching. However, only a few previous studies have addressed such re-switching of customers and unrealistic restrictive assumptions have been made.

This study deals with repetitive re-switching with the assumption of arbitrary number of stages and multi-period benefit maximization of consumers. Our model derives the range of initial market share which provides tenable equilibrium price. Such consideration of multi-stage equilibrium helps managers to make more meaningful market expectations. Our analysis also reveals that the condition of initial market share for tenable equilibrium includes all plausible results before the introduction of price discrimination.

By introducing the concept of repetitive

re-switching by consumers, this paper indicates that an increase in the discount factor or additional consideration of re-switching lowers a firm's incentive to discriminate prices. When the discount factor is one, firms lose the incentive to discriminate prices and the equilibrium prices correspond to those under the uniform price regime. In this way, prisoner's dilemma can be avoided. A few previous studies focused on the conditions in which prisoner's dilemma could be avoided; however, their investigation was limited to certain specific aspects like reward programs or asymmetries between firms. However, consumers' re-switching is a general aspect of poaching competition.

The reason why prisoner's dilemma can be avoided is that repetitive re-switching eliminates the incentive for firms to discriminate prices. Most peculiar aspects are that price discount for switchers which aims at attracting rival's consumers also makes its loyal consumers churn-out. This is because the loyal consumers can earn price discount in the next period by abandoning their preference in this period. Considering this, price discount has the aspect of lowering its market share as well as lowering its margin. Additionally, the acquired consumers have weak preference and generally seek lower price or discount. If discounted prices can only be acquired through switching, they will repetitively re-switch and can not be maintained. Therefore, price dis-

count for switchers is consumptive way of competition. Developing methods to lock-in their loyal consumers can give more profit in the long-run.

This consideration of re-switching influences, among other aspects, market shares and relative size of profits of both firms. Therefore, the results of previous studies that do not consider re-switching must be carefully interpreted. On the other hand, previous studies considering repetitive re-switching assumed that initial market share is the same as the equilibrium market share of uniform price regime. However, if firms are asymmetric and consumers maximize multi-period benefit, difference between brand qualities of firms should be lower for equilibrium to exist.

The evaluation of industry performance in this analysis is based only on profits. General linear-city models assume unit demand and high reservation price. Therefore, the total demand does not change according to the level of competition or price structure. Consequently, the change in social welfare only reflects the transportation cost or difference between the consumers' most preferred and purchased product. Therefore, the adoption of price discrimination changes the choices of consumers and social welfare; however, the impact varies according to the discount factor and the assumption of initial choices of consumers. In contrast, consumers' benefits generally increases when firms do not focus on re-switching

by consumers or future profit because the competition for poaching becomes intense.

Additionally, in the real world, most of subscription markets such as mobile telecommunication industry impose consumers the required subscription period for preventing them from churning out. However, in this model we do not consider the switching cost for consumers. As mentioned in the Introduction, the previous studies can be classified into two classes: assuming heterogeneous goods with no switching cost and assuming homogeneous goods with switching cost. It is because assuming heterogeneous goods and switching cost makes the model too complex to analyze. Combining these two will be helpful to understand price discrimination for churn-in.

## References

- Bester, H. and Petrakis, E. (1996), "Coupons and Oligopolistic Price Discrimination," *International Journal of Industrial Organization*, 14, 227-242.
- Chen, Y. (1997), "Paying Customers to Switch," *Journal of Economics and Management Strategy*, 6, 877-897.
- Corts, K. (1998), "Third-degree Price Discrimination in Oligopoly: All-out Competition and Strategic Commitment," *Rand Journal of Economics*, 29, 306-323.
- Fudenberg, D. and Tirole, J. (2000), "Customer Poaching and Brand Switching," *Rand Journal of Economics*, 31, 634-657.
- Holmes, T. J. (1989), "The Effects of Third-degree Price Discrimination in Oligopoly," *American Economic Review*, 79, 244-250.
- Kim, B.-D., Shi, M. and Srinivasan, K. (2001), "Reward Programs and Tacit Collusion," *Marketing Science*, 20, 99-120.
- Pazgal, A. and Soberman, D. (2008), "Behavior-based Discrimination: Is It a Winning Play, and If So, When?" *Management Science*, 27, 977-994.
- Shaffer, G. and Zhang, Z. J. (1995), "Competitive Coupon Targeting," *Marketing Science*, 14, 395-416.
- \_\_\_\_\_, \_\_\_\_\_. (2000), "Pay to Switch or Pay to Stay: Preference-based Price Discrimination in Markets with Switching Costs," *Journal of Economics and Management Strategy*, 9, 397-424.
- \_\_\_\_\_, \_\_\_\_\_. (2002), "Competitive One-to-One Promotions," *Management Science* 48, 1143-1160.
- Singh, S., Jain, D. and Krishnan, T. (2008), "Customer Loyalty Programs: Are They Profitable?," *Management Science*, 54, 1205-1211.
- Song, J. (2009), "Analysis of Economic Incentives for Price Discount Presupposing Churn-in," *Journal of the Korean Operations Research and Management Science Society*, 34, 55-75.
- \_\_\_\_\_, (2010), "Price Discrimination for Churn-in: Considering Asymmetric Firms and Consumers' Repetitive Re-switching," *Journal of Korean Marketing Association*, 25(4), 1-17.

- Taylor, C. (2003), "Supplier Surfing: Competition and Consumer Behavior in Subscription Markets," *Rand Journal of Economics*, 34, 223-246.
- Thisse, J. and Vives, X. (1988), "On the Strategic Choice of Spatial Price Policy," *American Economic Review*, 78, 122-137.
- Villas-Boas, M. J. (1999), "Dynamic Competition with Customer Recognition," *Rand Journal of Economics*, 30, 604-631.

## Appendix 1. Proof of Conditions in Proposition 1

The net benefits from each purchase schedule  $(i,j)$  should be compared to see the equilibrium condition. First, consider the choices of consumers in  $[0,x_0]$ .  $x_a$ ,  $x_b$ ,  $x_c$ , and  $x_3$  are defined as follows:

$$U_{(A,A)} - U_{(A,B)} = \delta(t - 2tx_a - u + P_b - P_a) = 0,$$

$$U_{(B,A)} - U_{(A,B)} = (1 - \delta)(-t + 2tx_b + u - P_b + P_a) + \delta(P_a - P_a) = 0,$$

$$U_{(A,A)} - U_{(B,B)} = (1 + \delta)(t - 2tx_c - u + P_b - P_a) + \delta(P_B - P_b) = 0,$$

$$U_{(B,A)} - U_{(B,B)} = \delta(t - 2tx_3 - u + P_B - P_a) = 0.$$

If  $U_{(A,A)} - U_{(A,B)}$  is negative,  $U_{(B,A)} - U_{(A,B)}$  is positive because the left term of  $U_{(B,A)} - U_{(A,B)}$  has always opposite sign of  $U_{(A,A)} - U_{(A,B)}$ . This means that  $U_{(A,B)}$  is smaller than  $U_{(A,A)}$  or  $U_{(B,A)}$ . Therefore,  $(A,B)$  cannot be chosen by consumers in  $[0,x_0]$ . Furthermore,  $x_c - x_1 = (\delta(1 + \delta)(P_a - P_a) + \delta(P_B - P_b)) / (2t(1 + \delta)) \geq 0$  and  $x_c$  is not binding. Therefore, only  $(B,B)$  and  $x_3 = (P_B - P_a + t - u) / (2t)$  should be considered in addition to  $\langle$ Figure 1 $\rangle$ . With similar calculation, with respect to consumers in  $[x_0,1]$ , only  $(A,A)$  and  $x_4 = (-P_a + P_b + t - u) / (2t)$  ( $U_{(A,B)}^{\text{Consumers} \in [x_0,1]} - U_{(A,A)}^{\text{Consumers} \in [x_0,1]} = \delta(-t + 2tx_4 + u + P_a - P_b) = 0$ ) should be additionally considered. Furthermore,  $x_4 \leq x_3$  is satisfied. Then, when  $x_0$  is between  $x_4^*$  and  $x_3^*$ ,  $(B,B)$  cannot be chosen by consumers in  $[0,x_0]$ , and  $(A,A)$  cannot be chosen by consumers in  $[x_0,1]$ . Therefore,  $x_4^* \leq x_0 \leq x_3^*$  is the necessary condition for  $P_i^*$ s to be the equilibrium. With equilibrium prices,  $x_4^* \leq x_0 \leq x_3^*$  is as follows:

$$((1 + \delta + 4\delta^2)t - (1 + \delta)u) / (4(t + 2\delta^2t)) \leq x_0 \leq ((3 - \delta + 4\delta^2)t - (1 + \delta)u) / (4(t + 2\delta^2t)).$$

On the other hand, only the case of  $P_A^* \geq P_a^*$  and  $P_B^* \geq P_b^*$  is being considered in this paper. To satisfy this condition,  $x_0$  should satisfy  $1/3 \leq x_0 \leq 2/3$ .

$x_0^U$  always satisfies  $1/3 \leq x_0^U \leq 2/3$  and higher than  $((1 + \delta + 4\delta^2)t - (1 + \delta)u) / (4(t + 2\delta^2t))$ . However,  $x_0^U$  is lower than  $((3 - \delta + 4\delta^2)t - (1 + \delta)u) / (4(t + 2\delta^2t))$  if and only if  $u$  satisfies  $u \leq 3t / (1 + 4\delta)$ .

## Appendix 2. Proof of Proposition 2

If  $\delta=1$ ,  $P_i^* = P_i^U$ , and if else,  $P_A^* < P_A^U$ ,  $P_a^* < P_a^U$ ,  $P_B^* < P_B^U$  and  $P_b^* < P_b^U$  are straightforward. The partial derivatives of the prices with respect to  $\delta$  are represented in the following manner:

$$\frac{\partial P_A^*}{\partial \delta} = \frac{4(1+(\delta^2+2\delta-2)x_0)t}{3(1+\delta)^2} > 0 \because x_0 < \frac{1}{2}, \quad \frac{\partial P_a^*}{\partial \delta} = \frac{4x_0t}{3} > 0,$$

$$\frac{\partial P_B^*}{\partial \delta} = \frac{4(\delta(2+\delta)(1-x_0)+2x_0-1)t}{3(1+\delta)^2}, \quad \frac{\partial P_b^*}{\partial \delta} = \frac{4(1-x_0)t}{3} > 0.$$

Because Firm  $B$ 's brand quality is higher than Firm  $A$ 's ( $u \geq 0$ ),  $x_0$  can not be higher than  $1/2$ .  $\partial P_B^*/\partial \delta$  is negative only when  $\delta$  has a lower than a certain value.

On the other hand, considering the equilibrium prices, the profit of firm  $A$  and  $B$  is represented in the following manner:

$$\Pi_A^* = \frac{u^2 - 2tu(2 + \delta - (1 - \delta)x_0) + t^2(5 + 2\delta + 2\delta^2 - 2(1 - \delta)(5 - 2\delta)x_0 + 10(1 - \delta)^2x_0^2)}{9t},$$

$$\Pi_B^* = \frac{u^2 + 2tu(1 + 2\delta + (1 - \delta)x_0) + t^2(5 - 4\delta + 8\delta^2 - 2(1 - \delta)(5 - 8\delta)x_0 + 10(1 - \delta)^2x_0^2)}{9t}.$$

The partial derivative of  $\Pi_i^*$  with respect to  $\delta$  is represented in the following manner:

$$\frac{\partial \Pi_A^*}{\partial \delta} = \frac{2(t(1 + 7x_0 - 10x_0^2 + 2\delta(1 - 2x_0 + 5x_0^2)) - u(1 + x_0))}{9},$$

$$\frac{\partial \Pi_B^*}{\partial \delta} = \frac{2(t(-2 + 13x_0 - 10x_0^2 + 2\delta(4 - 8x_0 + 5x_0^2)) + u(2 - x_0))}{9}.$$

Considering that the equilibrium  $x_0$  is lower than  $1/2$ , because  $2\delta(1 - 2x_0 + 5x_0^2) > 0$ ,  $\delta=0$  gives the lowest  $\partial \Pi_A^*/\partial \delta$ . Subsequently, considering that  $u < t$ ,  $\partial \Pi_A^*/\partial \delta|_{\delta=0} > 0$  and  $\partial \Pi_A^*/\partial \delta > 0$ . Next, because the equilibrium  $x_0$  is lower than  $1/2$ ,  $\partial^2 \Pi_B^*/\partial \delta^2 = 4t(4 - 8x_0 + 5x_0^2)/9 > 0$  and  $\delta=0$  provides the lowest  $\partial \Pi_B^*/\partial \delta$ . Furthermore,  $\partial^2 \Pi_B^*/\partial \delta \partial x_0|_{\delta=0} = 2(t(13 - 20x_0) - u)/9 > 0$ . Therefore,  $\delta=0$  and the lowest value of  $x_0$  ( $1/3$ ) provides the lowest  $\partial \Pi_B^*/\partial \delta$  and  $\partial \Pi_B^*/\partial \delta$  is positive.

## 소비자들의 반복적 전환가입을 고려한 가격차별 경쟁의 균형분석

송재도\* · 조은성\*\*

### 요 약

논문은 자사 가입자와 경쟁사 가입자에 대한 가격차별 경쟁상황을 다룬다. 이러한 경쟁에서 소비자들은 가입기업을 반복적으로 바꿈으로써 전환가입에 따른 할인을 추구할 수 있다. 그러나 기존 관련 문헌들은 이를 고려하지 못하거나 제한적인 가정 하에서 분석해 왔다. 본 논문에서는 임의의 기간 및 소비자들의 다기간 효용극대화를 가정하여 소비자들의 반복적 전환가입을 고려한 일반적인 균형을 도출하였으며, 다기간 동안 지속되는 가격균형을 도출하는 초기 시장점유율의 조건을 도출하였다. 이러한 다기간 동안 지속되는 균형만이 시장에 대한 합리적 예측으로 의미를 가질 수 있다. 또한 초기 시장점유율의 조건이 가격차별이 도입되기 전의 가능한 시장상황들을 포함하고 있음을 검토하였다. 도출된 균형에서 소비자들의 반복적 전환가입은 기업들의 경쟁기업 소비자의 획득 노력을 다음 기에 수포로 돌아가게 한다. 또한 전환 가입자에 대한 할인은 반복적 전환가입 상황에서 경쟁기업 가입자의 전환가입 뿐 아니라 자사 가입자에 대한 전환가입을 유발할 수 있다. 따라서 반복적 전환가입의 고려는 가격할인 경쟁을 완화하는 효과를 낳는다.

주제어: 가격차별, 제품차별화, 수평적차별화, 전환가입

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