Cost of Financial Distress on Financing Strategy of Callable Bonds*

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This paper theoretically explores the effect of cost of financial distress on the design and calling of a bond. A callable bond, in this paper, represents both callable non-convertible bonds and callable convertible bonds. This paper adopts a game model of the manager-shareholder and the bondholder, wher the former is a strategic player and the latter is a price-taker.

Regarding callable non-convertible debt, the model predicts that, in equilibrium, a firm defaults with the same probability as in the case of issuing the optimal standard bond. It implies that a callable non-convertible bond has no relative advantage over a standard bond when the cost of financial distress is considered.

However, the model provides an interesting implication regarding the timing of call that a call price applicable at a later time is relatively higher than a call price applicable for an earlier time as the cost of financial distress is larger. It implies that a firm should expect to pay relatively higher call price if it calls a bond later instead of calling right now. Thus, a firm is less likely defer calling because the marginal benefit of deferring is lower. This result can be explained by that a firm issues a callable bond to reduce the chance of financial distress. Calling a non-convertible bond tends to be expedited when the cost of financial distress is large. Calling entails recapitalization. By exercising a call option embedded in a bond, a firm repays outstanding debts and raise new funds. It tends to have a stronger incentive to exercise call options early and thus avoid financial distress since the cost of financial distress is larger.

On the other hand, the model shows that a firm remains bond-financed in a strictly larger set and defaults in a strictly smaller set of realized states in an intermediate period if it issues a callable convertible bond instead of a non-convertible bond. A firm is defined to remain bond-financed if a bond was issued in the beginning and has been neither called nor converted. Thus, a callable convertible bond enhances, if any, the benefit of bond-financing because a firm remains bond-financed with higher probability. It also mitigates cost of default because a firm defaults with lower probability. This result is driven by the design of a convertible bond. A convertible bond is designed to be converted if the return of a firm is relatively poor.

This implies that, if information asymmetry is incorporated, a relatively good firm may issue a callable non-convertible bond for signaling while a relatively poor one issues a callable convertible bond. A good firm can incur signaling cost by issuing a non-convertible bond because a non-convertible bond has relative disadvantage. A poor firm chooses to be separated by issuing a convertible bond if it is better off. It also explains the relatively poor performance of firms around a call of convertible debt reported by existing literature. This may be further interpreted as implying that a call conveys negative information.

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In equilibrium, cost of financial distress ambiguously influences a sequence of call prices contrary to a non-convertible bond. Thus, the theoretical prediction about the effect of cost of financial distress on calling a convertible bond is ambiguous. A firm need not hurry calling to recapitalize because it becomes equity-financed and, therefore, free from default risk once a convertible bond is forced to be called.

Key words: callable bond, convertible bond, cost of financial distress.

Ⅰ. Introduction

This paper explores the effect of cost of financial distress on the design and calling of a bond. Cost of financial distress is defined by the cost incurred during a restructuring and/or liquidation.

A callable bond represents both callable non-convertible bonds and callable convertible bonds. A call option provides the issuing firm an optional right to redeem the bond at a pre-determined call price before maturity. On the other hand, a conversion option provides the bondholder an optional right to convert the bond into equity according to a pre-determined conversion ratio. Conversion ratio is the number of shares a bond is exchanged into upon conversion.

A callable bond might be issued primarily to utilize favorable refunding opportunities in the future. A firm faces an incentive to issue a callable non-convertible bond when they expect the market interest rate to fall after issuance, and an incentive to issue a callable convertible bond when they expect stock prices to go up. By calling outstanding bonds, a firm refunds at a lower interest rate (callable non-convertible bonds and non-conversion forcing calls), or force bond-equity swap (conversion forcing calls).

However, future refunding may not be the only reason why a firm issues a callable bond. In the United States, firms relied on callable bonds more heavily during the late 1970s and early 1980s, when the market interest rate kept rising, than during the late 1980s and early 1990s, when the market interest rate kept falling. The one-year Treasury bill rate in the secondary market showed a uniform increase from 4.64% in December 1976 to 14.7% in August 1981. Thereafter, the market interest rate fluctuated, generally going down, but remained above 10% until September 1984. The one-year Treasury bill rate uniformly decreased from 9.93% in October 1984 to 3.33% in July 1993. According to Moody's Industrial Manuals, about 90% of the listed bonds issued from the late 1970s to

1982 were callable, whereas only about 60% of all bonds issued since 1983 have been callable.

This paper predicts that the cost of financial distress expedites calling a non-convertible bond. The sequence of call prices of a non-convertible bond is predicted to be more steeply increasing as the cost of financial distress is larger. Thus, a firm is less likely to defer calling a non-convertible bond because the marginal benefit of deferring is lower. On the other hand, this paper predicts that the effect of cost of financial distress on the call prices of a convertible bond is ambiguous. In addition, it predicts that, in equilibrium, a convertible bond is (designed to be) converted when the firm's return is relatively low.

The approach of this paper has a relative merit over many existing works. It explicitly constructs an equilibrium financing strategy, and is able to elicit how a firm determines the schedule of time-dependent call prices and optimal call policy. Although providing useful implications, some theoretical works fail to explicitly construct an equilibrium financing strategy. For instance, Harris and Raviv (1985) analyze calling a convertible bond separately from its design. In their model, a firm is assumed to have a callable convertible bond with call price zero so that it can always force a conversion. Neglecting the design of a financial instrument may impair the validity of the derived implications unless it is theoretically justified.

The rest of this paper is composed as follows. Section 2 reviews the relevant literature. Section 3 examines the effect of cost of financial distress on the design and calling of a bond. Section 4 is a conclusion.

II. Review of Literature

To explain the prevalence of callable debt, previous studies have investigated whether callable debt has a relative advantage over a non-callable non-convertible bond or straight equity. A non-callable non-convertible bond is called standard bond hereafter. Existing work suggests several potential advantages of a callable bond; For instance, a callable bond facilitates recapitalization, mitigates cost of financial distress and/or agency cost of debt, provides tax subsidy and signals private information.

2.1 The Literature of Callable Non-Convertible Debt

The literature suggests that the most prominent function of a callable bond is to facilitate recapitalization. In other words, in compendium to a non-convertible bond, a call provision facilitates issuing a new security. For instance, Pye (1966) views a call provision as a pure option to facilitate refunding when the market interest rate falls. Vu (1986) suggests that a call provision is included to facilitate removing restrictive financing covenants and, so, facilitates recapitalization. He finds that a firm pays higher call premium if calling removes a restrictive covenant. Narayanan and Lim (1989) also finds supporting evidence. A significantly higher proportion of callable zero-coupon bonds have restrictive covenants compared with non-callable bonds.

There are also secondary effects from the refunding opportunity provided by a callable non-convertible bond. Smith and Warner (1979) postulate that a call provision mitigates the agency cost of debt. When a firm is in debt, the manager who acts in the interests of the shareholders may attempt to extract bondholder wealth in favor of the shareholders. This action typically entails increasing the firm's risk, which does not necessarily lead to firm value maximization. Bondholders rationally expect this kind of managerial incentive distortion when evaluating the value of a bond at the time of purchase, which results in firm value reduction ultimately damaging the shareholder interests. A call provision mitigates this type of agency cost because it facilitates refunding before maturity. This is because the manager's action to reduce firm value is penalized later at the time of refunding. Further, Barnea *et al.* (1980) point out that, if it is possible to issue a callable bond with state-contingent call prices, a firm can eliminate financial distress risk and so the incentive problem of debt. A firm eliminates financial distress risk by exercising a call option in the realized states in which a firm would default otherwise.

Tax-deductibility of interest payment is another potential advantage of a callable bond. Because interest payment is tax-deductible, financial leverage provides a benefit (tax-subsidy). Another potential source of benefit of a callable bond is tax rate differential between individuals. Boyce and Kalotay (1979) explain the universal existence of a call provision, based on the tax differential between the borrowers and the lenders. They postulate that an agent in a higher tax bracket has a lower after-tax discount rate. Because a lender, who is likely to be an individual investor, is in general in a relatively lower tax bracket, prefers decreasing stream of interest payment. A borrower also prefers decreasing stream of expenditure. Therefore, both parties prefer a callable bond because the call price is non-increasing over time. In addition, Narayanan and Lim (1989) analyze the effect of tax rate on optimal call policy. Calling depends on tax rate because both interest payment and call premium are tax-deductible.

Issuing a (callable) bond may signal private information of the managers via a change in financial leverage. Increasing leverage is predicted to convey favorable information about a firm. In the signaling literature, a manager makes a decision to issue or call a bond in order to maximize his/her compensation. The compensation scheme is predetermined, and known to outside investors. Investors infer firm characteristic from a financial decision utilizing their knowledge about the compensation scheme. Ross (1977) postulates that, if default is costly, the managers of a relatively good firm increase financial leverage as a signal of firm type. A firm incurs a signaling cost because the probability of default increases with financial leverage. Robbins and Schatzberg (1986) postulate that a relatively good firm issues short-term debt (or a callable bond) while a poor firm issues equity if a manager is more risk-averse than both a shareholder and a bondholder.

Previous empirical work supports the signaling or tax effect of a callable bond. Vermaelen (1981) suggests that the stock price reacts positively after an increase in financial leverage resulting from common stock repurchase. Cornett and Travlos (1989) provide a similar evidence. A leverage-increasing transaction leads to an abnormal stock price increase while a leverage-decreasing transaction leads to an abnormal stock price decrease. Further, Vu (1986) suggests that a firm calls a bond to change financial leverage, possibly, for signaling purposes. He finds that the stock price reacts in the same direction as a change in financial leverage following a call.

2.2 The Literature of Callable Convertible Debt

A callable convertible bond is an equity-like as well as a debt-like issue. It can replicate the cash flows of issuing callable non-convertible bonds followed by those of issuing straight equity.

Stein (1992) suggests that a callable convertible bond is issued for deferred equity-financing under asymmetric information. A relatively good firm forces a conversion after private information is released. Further, Stein suggests that a good firm issues long-term debt to signal its good type while a poor firm issues callable convertible debt to reduce default risk. A good firm can manage risk arising from the issuing of a long-term bond, whereas a poor firm cannot.

Both Smith and Warner (1979) and Green (1984) postulate that callable convertible debt mitigates the aforementioned agency cost of debt more efficiently than callable non-convertible debt. As discussed earlier, the call option mitigates the agency cost of debt because it offers a refunding opportunity before maturity. The conversion option further mitigates the agency cost of debt because it offers bondholders the right to eventually be shareholders. In an equity-financed firm, shareholder wealth maximization complies with firm value maximization, thus avoiding the agency cost of debt. According to this view, a callable convertible bond is a debt-like issue.

Dann and Mikkelson (1984) find that common shareholders earn significantly negative abnormal returns both at the announcement of a convertible bond offering and at issuance. In contrast, the effect of a non-convertible bond offering is only marginally negative at the initial announcement and almost zero at issuance. They conclude that a convertible bond is an equity-like issue.

The optimal timing of call and conversion has also been studied in the literature. Brennan and Schwartz (1977) claim that it is optimal for a firm to call a convertible bond as soon as the conversion value exceeds the call price. Ingersoll (1977) finds that, in practice, calling is deferred long after the optimal timing predicted by Brennan and Schwartz (1977). Harris and Raviv (1985) attempt to resolve this apparent discrepancy using a signaling argument under a condition of information asymmetry. According to them, a good firm voluntarily chooses to defer calling to signal its good type, whereas a poor firm is less likely to defer calling because it has higher probability of being unable to force a conversion in a later stage.

Empirical studies support that calling a convertible bond is related with low performance of a firm. Lin and Chen (1991) find that stock prices show a negative reaction to the announcement of calling a convertible bond. Ofer and Natarajan (1987) document a drop in profitability upon calling a convertible bond.

III. Cost of Financial Distress on Financing Strategy of a Callable Bond

This section analyzes the effect of cost of financial distress on financing strategy of callable bonds. A firm is defined to be in financial distress when the value of existing total asset including future business opportunity falls short of the value of the total debt.

Restructuring/Liquidation is costly to a firm. Rationally expecting this contingency, a creditor properly incorporates the expected cost of financial distress in evaluating the value of a bond issue. Thus, it eventually costs the shareholders. Among sources of the cost, a restructuring or a liquidation normally incurs a considerable amount of administrative cost including legal cost. In addition, the resale value of asset must be accounted for if a restructuring or liquidation involves transactions of asset. The resale value of asset is likely to be lower than the marginal productivity in the production process of the original firm. Further, a bondholder may be different from a manager and/or a shareholder in a non-trivial way. After a restructuring, a former bondholder may not be so efficient in managing a firm or in monitoring a professional manager as an original shareholder. Thus, a restructured firm may incur larger managerial expense than otherwise.

This paper constructs a basic model of bond-financing in which a firm issues a standard bond. Then, it will be used as a benchmark to be compared with issuing either a callable non-convertible bond or a callable convertible bond.

3.1 The Basic Model

A basic model is constructed as follows. There is a firm held entirely by a manager-shareholder at time zero (t_0). The firm is endowed with an investment project that can run, at most, two periods. At t_0, the firm issues a standard bond to finance operating capital of the project for the first period. If the operating capital is fully funded, the project is implemented. The project generates a non-negative gross return at time one (t_1), denoted by y_1, following a certain probability distribution. Period 1 is defined as the time interval between t_0 and t_1. Upon observing the realized return of period 1, the manager-shareholder decides whether to hold the firm one more period or to default on outstanding debt. If the firm does not default, it funds the operating capital for the coming period. The project also generates a non-negative gross return at time two (t_2), denoted by y_2, following an identical and independent probability distribution as y_1. Period 2 is defined as the time interval between t_1 and t_2. The firm does not have an additional investment project available and, therefore, is liquidated at t_2.

The probability density function of gross return in one period is denoted by $z(y)$. Assume $z(y)>0$ on $[0, Y_{\max}]$ for some $Y_{\max}$, $<\infty$ and $z(y)=0$ otherwise. In addition, $z(y)$ is differentiable inside of the interval and its expected value $E(y)=R<\infty$. This probability density function of return is known to both the manager and outside investors.

The firm needs lump sum operating capital of amount K to run the project in one period. For simplicity, assume that it initially has no pecuniary capital. Thus, the firm raises a fund of amount K by issuing a bond at t_0. Assume $K<R$ so that the project can generate positive expected net return in each period. The firm behaves strategically in designing a bond. There is a representative outside investor who has fund sufficient to implement the project in period 1. She is a price-taker. The investor is willing to purchase

a bond at a price K if the expected gross return from the bond is no less than K. Assume that the risk-free market interest rate is zero.

For the time being, assume that the firm issues a standard bond maturing at t_2.[1] Provided the firm issues a bond and implements the project at t_0, it makes a decision to default after y_1 is realized at t_1. There are two possible paths of action at t_1. First, if the firm does not default at t_1, the firm funds the operating capital and runs the project in period 2. If y_1 exceeds K, the firm retains the amount K for the operating capital in the next period and pays the shareholder $y_1 - K$ as dividend. If y_1 falls short of K, the firm retains y_1, and issues a new straight bond maturing at t_2 to fund the difference, $K - y_1$. The firm is liquidated based on limited liability at t_2. Second, if the firm defaults on outstanding debt at t_1, the bondholder becomes the sole owner of the firm. She acquires the remaining total asset that is the sum of the realized return y_1 and the investment opportunity of period 2. The bondholder decides whether or not to restructure the firm. If the firm is not restructured, the bondholder recoups only y_1. If the firm is restructured, the bondholder recoups y_1 at t_1, funds and runs the project in period 2, and recoups the realized return y_2 at t_2.

Default is costly to a firm. For simplicity, assume that the firm incurs fixed cost of financial distress D only if it is restructured at t_1.[2] In addition, default costs a firm at t_1 but not at t_2. This does not affect the essence of the conclusion. It is adopted to compare a callable bond with a standard bond in terms of expected cost of financial distress. In the present model, a firm remains bond-financed in period 2 if it does not default at t_1. Thus, a firm possibly defaults at t_2. When a callable bond is subsequently incorporated into the model, a firm might be fully equity-financed and, thus, free from default-risk in period 2 if it calls a bond at t_1. This exaggerates the expected cost of financial distress of a standard bond if default were costly at t_2 as well.

In summary, the structure of the game is as follows. There are two players, namely, a manager-shareholder and a representative investor. The manager issues a standard bond with face value F to raise the required capital K at t_0. If the manager raises the required capital, he implements the project. The manager decides whether or not to default on the debt at t_1 after the return is

1) Thus, calling is irrelevant.
2) It does not affect the essence of the conclusion even if a firm incurs default cost regardless of the restructuring decision.

observed. If the firm defaults at t_1, the bondholder acquires the entire remaining assets of the firm and makes a decision to restructure. A strategy of the manger-shareholder consists of an action of designing a bond at t_0 and an action of default at t_1. The manager behaves strategically in designing a bond. The manager makes a decision to default as a function of the specification of an outstanding bond and the realized return at t_1. A strategy of the investor consists of an action of purchasing a bond at t_0 and an action of restructuring a defaulting firm at t_1. The investor is a price-taker in purchasing a bond. She purchases a bond at a price equal to the required capital K if the expected value is no less than K. The investor makes a decision to restructure as a function of the realized return at t_1 and the cost of restructuring.

A pure strategy of the firm (or the manager) is represented by the face value of a standard bond issued at t_0, F, and a decision to default at t_1, m. Formally, a pure strategy of the firm is denoted by $s = (F^S, m^S)$. The prescribed decision m^S is represented by a function $m^S : \{(F, y_1)\} \rightarrow \{M, NM\}$, where M stands for "to default" and NM stands for "not to default". A pure strategy of the investor consists of the decision to purchase a bond at t_0, l, and a decision to restructure at t_1, n. Formally, it is denoted by $u = (l^u, n^u)$. The prescribed decision to purchase is represented by a function $l^u : \{F\} \rightarrow \{L, NL\}$, where L stands for "to buy" and NL stands for "not to buy". The prescribed decision to restructure is represented by a function $n^u : \{m, D, R\} \rightarrow \{N, NN\}$, where N stands for a restructuring and NN stands for non-restructuring. This model only considers a pure strategy.[3] The strategy set of the firm is denoted by S and that of the investor is denoted by U.

The payoff of a player is defined as the expected net return as of t_0. The payoff function of the manager-shareholder is, given that his payoff is zero if the firm is not able to implement the project at t_0.

$$W^Z(s,u) = W^Z((F^s, m^S),(l^u, n^u))$$
$$= \begin{cases} 0 \ \text{ if } l^u(F^S) = NL \\ \Pr(m^S(F^S, y_1) = NM)\{E(y_1 | m^S(F^S, y_1) = NM) - \\ \qquad K + \Pr(y_2 \geq F^S)\, E(y_2 - F^S | y_2 \geq F^s)\} \\ \qquad \text{if } l^u(F^S) = L \end{cases}$$

The payoff function of the investor is, given that her payoff is zero if she does not purchase a bond at t_0.

3) Indeed, there does not exist mixed strategy equilibrium because pure strategy equilibrium is unique as shown subsequently.

$$W^i(s,u) = W^i((F^S, m^S),(l^u,n^u)) = \begin{cases} 0 \text{ if } l^u(F^S) = NL, \\ \Pr(m^S(F^S,y_1) = NM)\{\Pr(y_2 < F^S)E(y_2 \mid y_2 < F^S) \\ \quad + \Pr(y_2 \geq F^S)F^S\} + \Pr(m^S(F^S,y_1) \\ \quad = M)\{E(y_1 \mid m^S(F^S,y_1) = M) - K + R - D\} \\ \quad \text{if } l^u(F^S) = L \text{ and } n^u(m^S,D,R) = N) \\ \Pr(m^S(F^S,y_1) = NM)\{\Pr(y_2 < F^S)E(y_2 \mid y_2 < F^S) \\ \quad + \Pr(y_2 \geq F^S)F^S\} + \Pr(m^S(F^S,y_1) \\ \quad = M)E(y_1 \mid m^S(F^S,y_1) = M) \\ \quad \text{if } l^u(F^S) = L \text{ and } n^u(m^S,D,R) = NN \end{cases}$$

As the equilibrium concept, this paper adopts a sequential equilibrium. A sequential equilibrium coincides with a sub-game perfect equilibrium under perfect information. Sub-game perfection requires that, in equilibrium, the firm makes a decision of default at t_1 based on the value of equity at t_1. This value is whichever is greater, the gross benefit of default or that of not-to-default. The former is zero under limited liability. The latter is the sum of net dividend at t_1, $y_1 - K$, and the expected amount the shareholder recoups at t_2, $\Pr(y_2 \geq F)E(y_2 - F|y_2 \geq F)$. Then, the firm does not default if $y_1 \geq \delta$, where $\delta = -K + \Pr(y_2 \geq F)E(y_2 - F|y_2 \geq F)$, and defaults otherwise. In addition, sub-game perfection requires that the bondholder restructures a defaulting firm if the total benefit of restructuring is no less than that of non-restructuring and, otherwise, does not. The gross benefit of restructuring is $y_1 + (R - K) - D$, and that of non-restructuring is y_1. For comparative static analysis of the effect of cost of financial distress on financing strategy, this paper sets up the range of cost of financial distress so that a defaulting firm is always restructured. Formally, assume $D \leq R - K$.[4]

Definition 1. A sequential equilibrium of the above financing game consists of a pure strategy of the firm s^* and a pure strategy of the investor u^*. Formally, (s^*, u^*) satisfies

$$s^* = \underset{s \in S}{\operatorname{argmax}} \quad W^Z(s, u^*),$$
$$\text{and } u^* = \underset{u \in U}{\operatorname{argmax}} \quad W^i(s^*, u).$$

In addition, sub-game perfection requires

$$m^{s^*}(F^{s^*}, y_1) = \begin{cases} NM & \text{if } y_1 \geq \delta^* \\ M & \text{otherwise.} \end{cases},$$

$$\text{and } n^{u^*}(m^{s^*}, D, R) = \begin{cases} N & \text{if } m^{s^*} = NM \\ NN & \text{otherwise.} \end{cases}$$

Assume $R < 2K$. This condition is necessary for a firm to be unable to issue a default-risk-free standard bond. If $R \geq 2K$, a firm is default-risk free. A firm can issue a standard bond with a face value F such that the

4) Iit does not alter the essence of conclusion to assume that restructuring is never in the interest of the bondholder, i.e., $D > R - K$.

expected value of a bond equals K under limited liability, provided that a firm does not default at t_1. Face value F is chosen such that $\Pr(y_2 < F)E(y_2|y_2 < F) + \Pr(y_2 \geq F)F = K$. The bondholder is paid y_2 if $y_2 < F$, and F if $y_2 \geq F$ at t_2. With this bond, a firm never defaults at t_1 because the value of equity as of t_1 $(y_1 - K) + (R - K)$, is non-negative for all $y_1 \geq 0$.

3.2 The Optimal Strategy of a Standard Bond

As a preliminary step to investigate the relative advantage of a callable bond over a standard bond, this section establishes the optimal strategy of a standard bond. The sub-game perfect equilibrium of this game, as well as the games in the remainder of this section, can be expressed as the solution of an optimization problem of designing a bond at t_0. An optimization problem is set up assuming the following path of action. The firm designs a bond to sell at a price equal to the required operating capital of period 1, K. It implements the project after raising the required fund. A solution requires that the firm earns non-negative expected net return.[5] The optimization problem excludes a possibility of issuing a bond at a price higher than the required capital.[6] The optimization problem also excludes a possibility of raising a fund less than the required capital because the expected net return would be non-positive.

With regard to a standard bond, the problem is simply to choose a face value F.

$$\max \ W^Z = \Pr(y_1 \geq \delta)\{E(y_1 - K|y_1 \geq \delta) + \Pr(y_2 \geq F)E(y_2 - F|y_2 \geq F)\}$$

$$F \geq 0 \quad (1)$$

$$\text{s.t.} \ W^Z \geq 0 \quad (2)$$

$$\Pr(y_1 < \delta)E(y_1 + R - K - D|y_1 < \delta) + \Pr(y_1 \geq \delta)\{\Pr(y_2 < F)E(y_2|y_2 < F) + \Pr(y_2 \geq F)F\} \geq K \quad (3)$$

$$\delta - K + \Pr(y_2 \geq F)E(y_2 - F|y_2 \geq F) = 0 \quad (4)$$

The objective function expresses the expected net return of the manager-shareholder over period 1 and period 2, which is the expected return provided a firm does not default at time 1. Constraint (2) requires that the expected net return is non-negative. Constraint (3) states that the expected value of a bond should be no less than K in order to be marketable at a price K. The expected value of a bond incorporates firm decision to default in (sub-game perfect) equilibrium. The first term is the expected return pro-

5) In fact, a solution with positive expected net return exists as shown in the following.

6) This does not alter the conclusion if a firm is assumed to invest, if at all, a redundant amount over the required capital in a risk-free asset.

vided the firm defaults at t_1. The second term is the expected return provided the firm does not default at t_1. Cost of default is included in the expected return of the bondholder because she wholly owns a defaulting firm that bears the cost. Constraint (4) states that the net benefit of non-default is zero if $y_1 = \delta$. The firm does not default if $y_1 \geq \delta$, and defaults if $y_1 < \delta$.

Proposition 1. There exists unique solution F^d to this problem of designing a standard bond. In this equilibrium, the payoff of the manager-shareholder is the expected net return from the project over period 1 and period 2 minus the expected cost of default.

Proof. See Appendix 2.

3.3 The Optimal Strategy of a Callable Non-Convertible Bond

This sub-section further investigates whether a callable bond provides relative advantage over a standard bond when the cost of financial distress is considered. Assume that the firm issues a callable non-convertible bond with call prices, c_1 and c_2, and a face value *f*. For simplicity, assume that the firm may call a bond only at either t_1 after y_1 is realized or t_2 after y_2 is realized. The firm pays a call price c_1 if it calls the bond at t_1, and c_2 if it calls at t_2.

The firm simultaneously makes a decision to default and a decision to call at t_1. If the firm defaults, the bondholder becomes the sole owner of the firm. If the firm does not default but calls a bond, the shareholder recoups the realized return of period 1 and funds both the call price and the operating capital of the project for period 2. The shareholder is paid net amount, $y_1 - c_1 - K$, and becomes the sole claimant to the realized return of t_2. If the firm neither defaults nor calls, the shareholder recoups the realized return of period 1 and funds the project for period 2. The firm is liquidated based on limited liability at t_2. The firm chooses whether to call at a call price c_2, or repay at a face value *f* the bond at t_2 if the realized return y_2 is no less than $\min(c_2, f)$. The firm defaults otherwise.

The scope of equilibrium firm action at t_1 and t_2 can be narrowed because the game adopts a sub-game perfect equilibrium. First, sub-game perfection prescribes the firm action of redeeming a bond at t_2 given that the firm has neither defaulted nor called a bond at t_1. At t_2, the firm can choose whether to call the bond at a call price c_2 or repay it at a face value *f* if the realized return y_2 is no less than $\min(c_2, f)$, and defaults otherwise. Thus, the value of a bond and, thus, the action of a player depends only on $\min(c_2, f)$ in equilibrium. Without loss of generality, assume that $\min(c_2, f) = c_2$. Thus, the firm always calls a bond at t_2. If $\min(c_2, f) = f$, it suffices to replace c_2 with f in a solution.

Second, the prescribed firm action of defaulting and calling at t_1 depends on the specification of a bond issued at t_0 and the realized return y_1. In case $y_1 \geq \delta = \Delta(c_2)$, the firm certainly does not default and has a choice whether to call a bond or not. It makes the decision based on the relative net benefit of calling. The (relative) net benefit of calling as of t_1 is $\{(y_1 - K) - c_1 + R\} - \{(y_1 - K) + \Pr(y_2 \geq c_2)E(y_2 - c_2 | y_2 \geq c_2)\}$. The first term is the total benefit of calling, i.e., the sum of net dividend, (negative) call price and the total value of the project in period 2. The second term is the gross benefit of non- calling, i.e., the sum of net dividend at t_1 and the expected amount the shareholder recoups at t_2.[7] Then, calling is preferred if the call price is less than the value of uncalled bond, i.e., if $c_1 < R - \Pr(y_2 \geq c_2)E(y_2 - c_2 | y_2 \geq c_2) = \Pr(y_2 < c_2)E(y_2 | y_2 < c_2) + \Pr(y_2 \geq c_2)c_2$. On the other hand, in case $y_1 < \delta_C$, the firm defaults unless it calls a bond. The (relative) net benefit of calling equals the total benefit of calling, $(y_1 - K - c_1 + R)$, because the firm defaults if a bond is not called.[8] It follows that the firm calls a bond if the call price c_1 is no greater than the value of the firm at time 1, i.e., if $c_1 \leq y_1 + R - K$, and defaults otherwise.

The equilibrium firm action at time 1 and t_1, prescribed by sub-game perfection, is summarized as follows. First, the firm defaults at t_1 if $y_1 < \delta_C$ and $y_1 < c_1 - (R - K)$ or, equivalently, if $y_1 < \mu$, where

$$\begin{aligned} \mu &= M(c_1, c_2) \\ &= \min(c_1, \Pr(y_2 < c_2)E(y_2 | y_2 < c_2) \\ &\quad + \Pr(y_2 \geq c_2)c_2) - (R - K) \end{aligned} \quad (5)$$

Second, the firm calls a bond at t_1 if $y_1 \geq \mu$ and $c_1 < \Pr(y_2 < c_2)E(y_2 | y_2 < c_2) + \Pr(y_2 \geq c_2)c_2$. Third, the firm neither calls a bond nor defaults if $y_1 \geq \mu$ and $c_1 \geq \Pr(y_2 < c_2)E(y_2 | y_2 < c_2) + \Pr(y_2 \geq c_2)c_2$. A bond is, thus, called at a call price c_2 at t_2. For future reference, denote the value of a callable bond of non-defaulting firm as of t_1 by

$$\begin{aligned} \beta_C = B(c_1, c_2) = \min(c_1, \Pr(y_2 < c_2) \\ E(y_2 | y_2 < c_2) + \Pr(y_2 \geq c_2)c_2) \end{aligned} \quad (6)$$

Note that β_C is greater than the value of restructured firm, i.e., $\beta_C > y_1 + R - K - D$ for $y_1 < \mu$.

7) Assume that the firm does not call a bond if it is indifferent between a call and non-calling.
8) Assume that the firm does not default if it is indifferent between defaulting and non-defaulting.

The problem of designing a callable non-convertible bond is expressed as,

$$\max\ W^Z = \Pr(y_1 \ge \mu)\ E(y_1 - K - \beta_C + R \mid y_1 \ge \mu)$$
$$\beta_C \ge 0 \quad (7)$$
$$\text{s.t. } W^Z \ge 0 \quad (2)$$
$$\Pr(y_1 < \mu)\, E(y_1 + R - K - D \mid y_1 < \mu) + \Pr(y_1 \ge \mu)\, \beta_C \ge K \quad (8)$$
$$\mu = \beta_C - (R - K) \quad (6)$$

The objective function is the expected net return to the shareholder over period 1 and period 2. Constraint (8) states that the expected value of a bond should be no less than K. The first term is the expected return provided the firm defaults at t_1. The second term is the expected return provided the firm does not defaults at t_1. Constraint (6) states that the firm is indifferent between defaulting and non-defaulting at t_1 if the realized gross return at t_1, y_1, equals μ. The firm defaults if $y_1 < \mu$ and, otherwise, does not.

Proposition 2. There exists unique equilibrium for issuing a callable non-convertible bond (with non-state-contingent call prices). The equilibrium is essentially identical to that of issuing a standard bond. The firm defaults in an identical set of realized returns at t_1, the shareholder receives an identical expected payoff and the firm remains bond-financed in an identical set of realized returns at t_1. Finally, c_2 is relatively higher compared with c_1 in equilibrium as the cost of default D is larger, in the sense that $\frac{\partial(\partial c_2^* / \partial c_1^*)}{\partial D} > 0$.

Proof. See Appendix 3.

Proposition 2 implies that a callable non-convertible bond has no relative advantage over a standard bond when the cost of financial distress is considered. This is driven by the design of the bond. The manager minimizes the probability of financial distress or, equivalently, minimizes the value of a bond as of t_1. It turns out that, in equilibrium, a firm defaults with the same probability as in the case of issuing the optimal standard bond.[9)]

9) Barnea *et al.* (1980) point out that, if it is possible to issue a callable bond with state-contingent call prices, a firm can eliminate cost of financial distress. A call price is defined to be state-contingent if it is determined by a non-trivial function of realized return at time 1. The function is specified in a call provision at the time of issue.However, this paper precludes the possibility of issuinga callable bond with state-contingent call prices. This is not a restrictive assumption because a callable bond with state-contingent call prices is rarely observed in practice. It may not be optimal to make this kind of contract because of the considerable cost of writing and enforcing such a contract. Specifically, it costs the parties of contract to verify the realized return and a callable bond with state-contingent call prices need specify the accounting method at the time of issue. However, it may benefit both parties to allow a firm to choose accounting method after the issuance instead of imposing a particular one.

It provides an interesting implication regarding the timing of call that a call price applicable at a later time is relatively higher than a call price applicable for an earlier time as the cost of financial distress is larger. It implies that a firm should expect to pay relatively higher call price if it calls a bond later instead of calling right now. Thus, a firm is less likely defer calling because the marginal benefit of deferring is lower.

This result can be explained by that a firm issues a callable bond to reduce the chance of financial distress. Calling a non-convertible bond tends to be expedited when the cost of financial distress is large. Calling entails recapitalization. By exercising a call option embedded in a bond, firms repay outstanding debts and raise new funds. They tend to have a stronger incentive to exercise call options early and thus avoid financial distress since the cost of financial distress is larger.

3.4 The Optimal Strategy of a Callable Convertible Bond

The following analysis shows that a callable convertible bond mitigates the expected cost of financial distress compared with a standard or a callable non-convertible bond. The model is extended to analyze a strategy of a callable convertible bond. The firm issues a callable convertible bond at time 0 with call prices v_1 and v_2, and a dilution factor e.[10] The dilution factor is the proportion of total equity the holder of a convertible bond acquires upon a conversion. Assume that a convertible bond may be converted after the realization of return of period 1 at t_1 only. It is for simplicity not to allow a conversion at t_0 because the preferences of the manager is subsequently specified such that a bond is not converted at t_0 in equilibrium. This model excludes conversion at time 0 to focus on strategy of borrowing. Recall that no new information is released until t_1. A bond is essentially straight equity if it is converted at t_0. Further, this model does not explicitly incorporate the action of converting after the realization of return of period 2 at t_2 because a sub-game perfect equilibrium action is prescribed in a straightforward manner at the end of the game; the realized value of a callable convertible bond at t_2 is fully accounted in terms of expected value in the decisions to call and to convert in earlier periods. In addition, it does not affect a solution whether a firm makes a decision to call before or after a bondholder makes a decision to convert. In a sub-game perfect equilibrium, a firm always calls a bond if the call price is less than the value of uncalled bond. And, a bondholder always converts a bond if the

10) Assume that face valu $f_v \geq v_2$.

conversion value is greater than the minimum of the call price and the value of uncalled bond.

The firm simultaneously makes a decision to default and a decision to call at t_1, provided that a callable convertible bond is not converted at t_0. If the firm defaults on outstanding debt, the bondholder becomes the sole owner of the firm. If the firm does not default at t_1, it makes a decision to call and the bondholder makes a decision to convert. There are three possible paths of action. First, in case a callable convertible bond is neither called nor converted, the original shareholder recoups the realized return of period 1 and funds the project for period 2. The firm is liquidated at t_2 based on limited liability. Second, in case a bond is called but not converted, the original shareholder recoups the realized return of period 1, and funds both the call price and the project for period 2. The shareholder becomes the sole claimant of the firm. Third, in case a bond is converted, the new shareholder has claim only to the dividend of t_2. At t_1, the original shareholder is paid net dividend, $y_1 - K$, and funds the project for period 2. At t_2, both the original and the new shareholder are paid a dividend according to their proportions of total equity.

This model assumes that the new shareholder is paid a dividend only at t_2 if the bond is converted at t_1, for two reasons. First, it eliminates the possibility of a conversion option being a (partial) substitute of state-contingent call prices. If conversion value depends on realized return of t_1, a conversion option (partially) replicates the effect of state-contingent call price because conversion value is a kind of redemption price of a bond. Second, it accentuates the function of a convertible bond as a debt issue. Alternatively, the model can be re-specified such that the new shareholder is paid dividend at both t_1 and t_2 regardless when a bond is converted. In this instance, a convertible bond is effectively an equity issue. However, it is preferred for a convertible bond to have an aspect of debt because this analysis presumes that there is certain benefit of bond-financing despite non-trivial cost if default. In addition, it is possible to re-specify the model such that the initial shareholder can choose upon a conversion whether or not to fund the project for period 2. However, the solution is essentially identical as shown subsequently.

A certain restriction on the manager's preferences is adopted to preclude mitigating default-risk by designing a callable convertible bond close to straight equity.[11] For instance, a firm is free from default-risk if a callable

11) This analysis presumes that there is certain benefit of bond-financing which dominates over non-trivial cost of default.

convertible bond is deemed to be converted in all realized states at t_1. Then, the convertible bond is essentially equivalent to equity issue. Thus, the manager preferences are restricted in a certain manner to prevent a convertible bond that is designed to emulate equity-financing. Specifically, the manager-shareholder strictly prefers to remain bond-financed without defaulting in a strictly larger set of realized y_1. If more than one strategy prescribes an identical set of realized returns in which a firm remains bond-financed, the manager-shareholder strictly prefers a one with strictly larger expected payoff.[12] In addition, a criterion on equilibrium is adopted to prevent designing a convertible bond inferior to a non-convertible bond in terms of the expected return to the manager. Specifically, in equilibrium, the expected payoff of the shareholder should be no less than that from issuing the optimal callable non-convertible bond. Thus, a callable convertible bond is, presumably, designed to be converted in a non-degenerate set of realized returns at t_1 in equilibrium. Otherwise, a callable convertible bond is essentially equivalent to and has no relative advantage over a callable non-convertible bond. Further, a callable convertible bond is not designed to be converted in all realized states at t_1 in equilibrium because, otherwise, the firm does not remain bond-financed in any realized state.

Sub-game perfection further narrows the scope of equilibrium strategy as described in Lemma 1.

Lemma 1. In equilibrium, for some value η, $0 < \eta \le \mu_v = M(v_1, v_2)$, a conversion is strictly preferred at time 1 if $y_1 < \eta$, weakly preferred if $y_1 = \eta$, and the firm remains bond-financed without defaulting if $y_1 \ge \eta_v \cdot \eta$ is expressed as $\eta = e \times R - (R - K - D)$.

Proof. See Appendix 4.

An equilibrium is characterized by the solution of the following problem of designing a callable convertible bond, equation (9) through (14).

$$\max \quad W^Z = \Pr(y_1 \le \eta^l)(1-e^l)R + \Pr(y_1 \ge \mu_v^{\,l})\,\beta_v^{\,l}.$$

$$\beta_v^{\,l}, e^l \in V_l \tag{9}$$

Define $V = \{\beta_v, e\} \mid (\beta_v, e) \ge (0,0), (\beta_v, e)$ is a solution for the following system, equation (10), (11), (12), (13) and (14)}. V_l is defined as the set of (β_v, e) of which β_v is the minimum among all the elements of V. Formally, $V_l = \{(\beta_v^{\,l}, e^l) \mid (\beta_v^{\,l}, e^l) = \text{argmin}\ \mu_v$ s.t. $(\beta_v, e) \in \text{V}\}$. A solution is constrained by

12) It is unnecessary to specify the preferences in further detail

the set V_l to maximize the set of realized states in which the firm remains bond-financed.

$$W^Z \geq W^Z(c_1^*, c_2^*)$$
$$= 2(R-K) - \Pr(y_1 < \mu^d)D \quad (10)$$
$$\Pr(y_1 \geq \eta)e \times R + \Pr(\eta < y_1 < \mu_v)$$
$$E(y_1 + R - K - D \mid \eta < y_1 < \mu_v)$$
$$+ \Pr(y_1 \geq \mu_v)\beta_v \geq K \quad (11)$$
$$\beta_v \geq e \times R \quad (12)$$
$$\mu_v = M(v_1, v_2) = \beta_v - (R-K) \quad (13)$$
$$\eta = e \times R - (R-K-D). \quad (14)$$

The objective function (9) expresses the expected payoff to the shareholder. The first term is the expected payoff to the shareholder provided a bond is converted when $y_1 \leq \eta$ at time 1. The second term is the expected payoff provided a firm does not default and a bond is not converted when $y_1 \leq \mu_v$. The firm is to default when $\eta < y_1 < \mu_v$. The inequality (10) requires that the expected payoff to the shareholder is no less than that from issuing the (optimal) callable non-convertible bond. The inequality (11) states that the value of a bond as of time 0 should be no less than the principal amount of borrowing K. The first term of the left-hand side is the expected payoff to the bondholder provided the bond is converted at t_1. The bond is converted if the conversion value exceeds the value of restructured firm, i.e., if $y_1 \leq \eta = e \times R - (R-K-D)$. The second term is the expected payoff provided a firm defaults at t_1, i.e., when $\eta < y_1 < \mu_v$.[13] The third term is the expected payoff provided a firm does not default and a bond is not converted at t_1, i.e., when $y_1 \geq \eta_v$. The inequality (12) states that the conversion value is no more than the value of non-converted bond as of t_1 provided $y_1 \geq \mu_v$. This is required for the firm to remain bond-financed in a non-degenerate set of realized states at t_1. The equation (13), which is adapted from equation (5) and (6), defines μ_v. The equation (14) defines η.

Proposition 3. There exists a unique equilibrium strategy for a callable convertible bond. In this equilibrium, a firm defaults in a strictly smaller set of realized states, and so mitigates cost of financial distress. The effect of cost of financial distress on the call prices of a convertible bond is ambiguous in the sense that the sign of $\partial \frac{(\partial v_2^* / \partial v_1^*)}{\partial D}$ is indeterminate in equilibrium.

Proof. See Appendix 5.

In equilibrium, the firm action at time 1 is

13) In a solution, this second term may be degenerate, i.e., $\eta = \mu_v$

as follows. If $y_1 \le \eta^d = e^d \times R - (R - K - D) < \mu_v{}^d$, the bond is converted. If $\eta^d < y_1 < \mu_v{}^d$, the firm defaults. The firm is not willing to call the bond, and the bondholder does not convert because the conversion value is less than the value of restructured firm. If, however, the solution implies $\eta^d = \mu_v{}^d$, the firm never defaults. If $y_1 \ge \mu_v{}^d$, the firm does not default and remains bond-financed. The bond is to be redeemed based on limited liability at t_2 because it is neither called nor converted at t_1.

Proposition 3 is driven by the design of a convertible bond. A convertible bond is designed to be converted if the return of a firm is relatively poor and so the conversion value exceeds the value of restructured firm, i.e., when $y_1 \le \eta^d$. Thus, the call prices can be set lower than those of the optimal callable non-convertible bond. This reduces the set of realized states in which the firm defaults.

This implies that, if information asymmetry is incorporated, a relatively good firm may issue a callable non-convertible bond for signaling while a relatively poor one issues a callable convertible bond. A good firm can incur signaling cost by issuing a non-convertible bond because a non-convertible bond has relative disadvantage. A poor firm chooses to be separated by issuing a convertible bond if it is better off.

It also explains the relatively poor performance of firms around a call of convertible debt observed by Lin and Chen (1991) and Ofer and Natarajan (1987). This may be further interpreted as implying that a call conveys negative information.

In equilibrium, cost of financial distress ambiguously influences a sequence of call prices contrary to a non-convertible bond. Thus, the theoretical prediction about the effect of cost of financial distress on calling a convertible bond is ambiguous. A firm need not hurry calling to recapitalize because it becomes equity-financed and, therefore, free from default risk once a convertible bond is forced to be called.

Ⅳ. Conclusion

This paper explores the effect of cost of financial distress on the design and calling of a bond. It predicts that the sequence of call prices of a non-convertible bond is predicted to be more steeply increasing as the cost of financial distress is larger. Thus, a firm is less likely to defer calling a non-convertible bond because the marginal benefit of deferring is lower. On the other hand, a firm can mitigate cost of financial distress by issuing a callable bond compared with non-convertible debt. In equilibrium, a convertible bond is (designed to be) converted when the return of a firm is relatively low. In addition, the effect of cost of financial distress on the call prices of a convertible bond is ambiguous.

However, these implications might depend on the assumption that a firm is fully equity-financed for period 2 if it calls a bond at t_1. This means that a firm is free from default-risk once it calls outstanding debt. It remains for further research what if a firm funds the call price by borrowing.

REFERENCES

Barnea, Amir, Robert A. Haugen, and Lemma W. Senbet (1980), "A Rationale for Debt Maturity Structure and Call Provisions in the Agency Theoretic Framework," *Journal of Finance* 35, pp. 1223-34.

Boyce, W.M. and A. J. Kalotay (1979), "Tax Differentials and Callable Bonds," *Journal of Finance* 34, pp. 825-38.

Brennan, M. and E. Schwartz (1977), "Convertible Bonds: Valuation and Optimal Strategies for Call and Conversion," *Journal of Finance* 32 pp. 1699-715.

Cornett, Marcia Millon and Nickolaos G. Travlos (1989), "Information Effects Associated with Debt-for-Equity and Equity-for-Debt Exchange Offers," *Journal of Finance* 44, pp. 451-68.

Dann, Larry Y. and Wayne H. Mikkelson (1984), "Convertible Debt Issuance, Capital Structure Change and Financial-Related Information: Some New Evidence," *Journal of Financial Economics* 13, pp. 157-86.

Fischer, Edwin O., Robert Heinkel, and Josef Zechner, "Dynamic Recapitalization Policies and the Role of Call Premia and Issue Discounts (1989)," *Journal of Financial and Quantitative Analysis* 24, pp. 427-46.

Gourieroux, C., A. Monfort and A. Trognon (1984), "Pseudo Maximum Likelihood Methods: Theory," *Econometrica* 52, pp. 681-700.

Green, Richard C (1984), "Investment Incentives, Debt and Warrants," *Journal of Financial Economics* 13, pp. 115-36.

Greene, William H. 1994, Econometric Analysis, the Second Edition, New York: Macmillan Publishing Company, pp. 715-27.

Harris, M. and A. Raviv (1985), "A Sequential Signalling Model of Convertible Debt Call Policy," *Journal of Finance* 40, pp. 1263-82.

Ingersoll, J. E., Jr. (1977), "An Examination of Corporate Call Policies on Convertible Securities," *Journal of Finance* 32, pp. 289-322.

Jaggia, Sanjiv (1991), "Tests of Moment Restrictions in Parametric Duration Models," *Economics Letters* 37, pp. 35-8.

Kreps, David M. and Robert Wilson (1982), "Sequential Equilibria," Econometrica 50, pp. 863-94.

Lin, Ji-Chai and K. C. Chen (1991), "Partially Anticipated Convertible Calls," *The Financial Review* 26, pp. 501-15.

Myers, Stewart C. and Nicholas S. Majluf (1984), "Corporate Financing and Investment Decisions When Firms Have Information that Investors Do not Have," *Journal of*

Financial Economics 13, pp. 187-221.

Narayanan, M.P. and Suk-Pil Lim (1989), "On the Call Provision in Corporate Zero-Coupon Bonds," *Journal of Financial and Quantitative Analysis* 24, pp. 91-103.

Ofer, Aharon R. and Ashok Natarajan. (1987), "Convertible Call Policies: An Empirical Analysis of an Information-Signalling Hypothesis," *Journal of Financial Economics* 19, pp. 91-108.

Pye, Gordon (1966), "The Value of the Call Option on a Bond," *Journal of Political Economy* 74, pp. 200-5.

Robbins, Edward Henry and John D. Schatzberg (1986), "Callable Bonds: A Risk-Reducing Signalling Mechanism," *Journal of Finance* 41, pp. 935-49.

Smith, Clifford M. and Jerold B. Warner (1979), "On Financial Contracting: An Analysis of Bond Covenants." *Journal of Financial Economics* 7, pp. 117-61.

Stein, Jeremy C (1992) "Convertible Bonds as Backdoor Equity Financing," *Journal of Financial Economics* 32, pp. 3-21.

Thatcher, Janet S (1985), "The Choice of Call Provision Terms: Evidence of the Existence of Agency Costs of Debt," *Journal of Finance* 40, pp. 549-61.

Titman, Sheridan and Reberto Wessels (1988), "The Determinants of Capital Structure Choice," *Journal of Finance* 43, pp. 1-19.

Vu, Joseph D (1986), "An Empirical Investigations of Calls of Non-Convertible Bonds," *Journal of Financial Economics* 16, pp. 235-65.

APPENDIX

Appendix 1. Table of Notation

Symbol	Description
t_i	time point i, i = 0, 1, 2
y_i	return of the firm at t_i, i = 1, 2
F	face value of a bond issued at t_0
m	firm's decision to default at t_i
l	Investor's decision to purchase a bond at t_0
n	Investor's decision to purchase a bond at t_0
s	pure strategy of the firm
u	pure strategy of the investor
R	expected value of the project
K	required capital for the project
D	fixed cost of financial cost
δ	cut-off level of return for default

Appendix 2. Proof of Proposition 1

The solution of equation (4) is a continuous and differentiable function, $\delta = \Delta(F)$,

$$\delta = \Delta(F) = K - \Pr(y_2 \geq F) E(y_2 - F | y_2 \geq F), \text{ and } \Delta' > 0. \quad (4')$$

To prove this, note that a solution, $\delta = \Delta(F)$, exists because the right hand side of equation (4′) is finite. Note that $\Delta(F)$ is strictly increasing in F. $\Delta(F)$ is continuous and differentiable by the assumption about the underlying probability distribution of return. Thus, the partial derivative $\Delta'(F) > 0$. Equation (3′) follows from substituting $\delta = \Delta(F)$ into constraint (3).

$$\Pr(y_1 < \Delta(F))E(y_1 + R - K - D \mid y_1 < \Delta(F)) + \Pr(y_1 \geq \Delta(F))\{\Pr(y_2 < F)E(y_2 \mid y_2 < F) + \Pr(y_2 \geq F)F\} \geq K. \quad (3')$$

A solution to the objective function (1) subject to constraint (3′) is unique because the objective function is strictly decreasing in F. (Thus, constraint (3′) holds with equality in a solution because the left-hand side of inequality (3′) is strictly increasing in F.) Assume that there does exist a solution Fd which also satisfies constraint (2). Otherwise, the problem is meaningless because the project is not worthwhile to implement due to the burden of the cost of default (as long as the firm issues a standard bond). In equilibrium, the expected payoff of the manager-shareholder fully accounts for the net benefit of the investment opportunity including the expected cost of default. Formally, the payoff of the manager-shareholder is

$$W^Z(F^d) = 2(R-K) - \Pr(y_1 < \delta^d)D,$$

where

$$\delta^d = K - \Pr(y_2 \geq F^d)E(y_2 - F^d \mid y_2 \geq F^d).$$

The expected value of the bond as of time 0 is expressed as,

$$\Pr(y_1 < \delta^d)E(y_1 + R - K - D \mid y_1 < \delta^d) + \Pr(y_1 \geq \delta^d)\{\Pr(y_2 < F^d)E(y_2 \mid y_2 < F^d) + \Pr(y_2 \geq F^d)F^d\} = K.$$

Appendix 3. Proof of Proposition 2.

A solution β_C, if at all, is unique because the objective function (7) strictly decreases in β_C (and μ strictly increases in β_C). From a comparison of this problem with the problem of designing a standard bond, it follows that there is a unique solution $\beta_C{}^d$ which meets constraint (2), constraint (8) with equality and constraint (6), where $\beta_C{}^d = \Pr(y_2 < F^d)E(y_2 \mid y_2 < F^d) + \Pr(y_2 \geq F^d)F^d$.

The solution is essentially identical to that of designing a standard bond. In particular, the firm defaults in an identical set of realized states, $\{y_1 \mid y_1 < \mu^d\}$ where $\mu^d = \beta_C{}^d - (R-K) = \delta^d$. Assume that a call price is set to equal the minimum value as long as the implied strategy is essentially identical. Then, from equation (6), it follows that the firm issues a callable non-convertible bond with call prices $c_1{}^d$ and $c_2{}^d$ (and a face value f^d) such that $c_1{}^d = \Pr(y_2 < c_2{}^d)E(y_2 \mid y_2 < c_2{}^d) + \Pr(y_2 \geq c_2{}^d)c_2{}^d$ and $c_2{}^d = F^d$, (and $f^d \geq c_2{}^d$). The firm does not call the bond and remains bond-financed in a set of realized states, $\{y_1 \mid y_1 \geq \mu^d\}$. In equilibrium, the expected payoff of the shareholder is identical with that of the standard bond because the firm defaults with the identical probability. Formally, $W^Z(c_1{}^d, c_2{}^d) = 2(R-K) - \Pr(y_1 < \mu^d)D = 2(R-K) - \Pr(y_1 < \delta^d)D = W^Z(F^d)$. The expected value of the bond as of t_0 is expressed as,

$$\Pr(y_1 < \mu^d)E(y_1 + R - K - D \mid y_1 < \mu^d) + \Pr(y_1 \geq \mu^d)\beta_c{}^d = K. \qquad (8')$$

In equilibrium, $c_1 = \int_0^{c_2} y\, z(y)dy + c_2 \int_{c_2}^{\infty} z(y)dy$. Thus, $\partial c_1{}^d / \partial c_2{}^d = \int_{c_2}^{\infty} d^Z(y)dy$.

Then, $\frac{\partial(\partial c_1^d / \partial c_2^d)}{\partial D} = -z(c_2^d)(\frac{\partial c_2^d}{\partial D}) < 0$ because $(\frac{\partial c_2^d}{\partial D}) > 0$. Thus, the sequence of call prices is more steeply increasing as cost of default, D, is larger, i.e., $\frac{\partial(\partial c_2^d / \partial c_1^d)}{\partial D} > 0$.

Appendix 4. Proof of Lemma 1.

Suppose that a convertible bond is issued at time 0. Then, the firm makes a decision to default and a decision to call at time 1. Refer to equation (5) and note that the firm defaults unless a bond is converted if $y_1 < \mu_v = M(v_1, v_2) = \beta_v - (R-K)$, where $\beta_v = B(v_1, v_2) = \min(v_1, \Pr(y_2 < v_2) E(y_2 \mid y_2 < v_2) + \Pr(y_2 \geq v_2) v_2)$. β_v is the value of a callable convertible bond as of t_1 provided it is not converted and the firm does not default at t_1. Note that the conversion value of a callable convertible bond as of t_1 is $e \times R$ for all $y_1 \geq 0$ because a new shareholder is paid a dividend according to his portion of equity, e, at t_2.

If a conversion is preferred for some realized return $y_1 = \eta'$ at t_1, a conversion is preferred for all realized return no greater than η'. To prove this, first, consider the possibility $\eta' < \mu_v$. This implies that the conversion value is no less than the value of restructured firm if $y_1 = \eta'$. Since the value of restructured firm strictly increases in y_1 for $y_1 < \mu_v$, a conversion is strictly preferred for all $y_1 < \eta'$. Second, if $\eta' \geq \mu_v$, the conversion value is no less than β_v. Then, a conversion is preferred for all $y_1 \geq \mu_v$. Since the value of restructured firm is strictly less than β_v, for all $y_1 < \mu_v$, a conversion is strictly preferred for all $y1 < \mu_v$ as well. Thus, a conversion is preferred for all $y_1 < \mu'$.

Recall that a callable convertible bond should be converted at t_1 in a non-degenerate set of realized y_1 and that the firm should remain bond-financed at t_1 in a non-degenerate set of realized y_1. Thus, a callable convertible bond must be designed such that the a conversion is strictly preferred for $y_1 < \eta \leq \mu_v$(i.e., $e \times R > y_1 + R - K - D$ for $y_1 < \eta \leq \mu_v$) and weakly preferred for $y_1 = \eta$, and that the firm remains bond-financed without defaulting for $y_1 \geq \mu_v$(i.e., $e \times R \leq \beta_v$ for $y_1 \geq \mu_v$). It follows $\eta = e \times R - (R - K - D)$.

Appendix 5. Proof of Proposition 3.

There exists unique solution to the optimization problem. To prove uniqueness, suppose there are solutions (β_v^1, e^1) and (β_v^2, e^2). Obviously, $\beta_v^1 = \beta_v^2$ by the assumed preferences. Also, $e^1 = e^2$ because equation (9) strictly decreases in e for any given β_v. To prove existence, the set V_l is shown to be non-empty and compact. Suppose that the firm issues a callable convertible bond with call prices, v_1' and v_2', and a dilution factor e'.

Define $e' = (R-K-D)/R + \varepsilon < K/(2R-K)$, $\varepsilon > 0$. (The inequality $(R-K-D)/R < K/(2R-K)$ holds under the assumption $R < 2K$. If $R < 2K$, the firm defaults in a non-degenerate set of realized states if it issues a non-convertible bond.) Denote $\beta_V' = B(v_1', v_2')$, $\mu_v' = \beta_v' - (R-K)$ and $\eta' = e' \times R - (R-K-D)$. Rewrite equation (8′) as

$$\Pr(y_1 \le \varepsilon \times R)\, E(y_1 + R - K - D \mid y_1 \le \varepsilon \times R) + \Pr(\varepsilon \times R < y_1 < \mu^d)\, E(y_1 + R - K - D \mid \varepsilon \times R < y_1 < \mu^d) + \Pr(y_1 \ge \mu^d)\beta_c^d = K. \qquad (8'')$$

Note that the left-hand side of equation (8″) is continuous and strictly increasing in both ε and β_C^d. Compare equation (8″) with inequality (11). Because $e' \times R > E(y_1 + R - K - D \mid y_1 \le \varepsilon \times R)$, there exists a pair($e'$, μ_v') such that $\varepsilon > 0$ and $\mu_v' < \mu^d$, which satisfies the weak inequality (11) with equality. Formally,

$$\Pr(y_1 \le \varepsilon \times R)\, e' \times R + \Pr(\varepsilon \times R < y_1 < \mu_v')\, E(y_1 + R - K - D \mid \varepsilon \times R < y_1 \le \mu_v') + \Pr(y_1 \ge \mu_v')\beta_v' = K. \qquad (11')$$

For a sufficiently small ε, $e' \times R < \beta_v' < \beta_c^d$ holds because $\beta_c^d > R - K - D$. If this callable convertible bond is issued, equations (10), (11), (12), (13) and (14) are satisfied. Thus, the set V is non-empty. It can be shown that the set V is compact. It follows that the set Vl is also non-empty and compact. Therefore, there exists a unique solution, denoted by (β_v^d, e^d), for the system specified in equations (9) through (14).

In equilibrium, the firm issues a callable convertible bond with call prices such that $v_1^d = \Pr(y_2 < v_2^d)\, E(y_2 \mid y_2 < v_2^d) + \Pr(y_2 \ge v_2^d)\, v_2^d = \beta_v^d$, and a dilution factor e^d. The firm remains bond-financed in a set of realized states $\{y_1 \mid y_1 \ge \mu_v^d\}$, $\mu_v^d < \mu^d$. The bond is converted in a set of realized states $\{y_1 \mid y_1 \le \eta^d\}$, $\eta^d = e^d \times R - (R-K-D) \le \mu_v^d$. The firm defaults in a set of realized states, $\{y_1 \mid \eta^d < y_1 < \mu_v^d\}$. The expected payoff to the shareholder is $W^Z = 2(R-K) - \Pr(\eta^d < y_1 < \mu_v^d)D > W^Z(c_1^d, c_2^d) = 2(R-K) - \Pr(y_1 < \mu^d)D$. The expected payoff to the bondholder is expressed as:

$$\Pr(y_1 \le \eta^d)e^d \times R + \Pr(\eta^d < y_1 < \mu_v^d)\, E(y_1 + R - K - D \mid \eta^d < y_1 < \mu_v^d) + \Pr(y_1 \ge \mu_v^d)\beta_v^d \ge K. \qquad (11'')$$

Further investigation of a solution shows whether (11″) is met with equality or strict inequality depends on parameters. If (11″) is met with strict inequality, the firm can sell the bond at a price higher than K and invest the amount over the required capital in a risk-free asset.

재무적 곤경 비용을 고려한 수의상환 채권의 재무전략

위정범*

요 약

이 논문은 기업의 재무적 곤경 비용이 수의상환 채권의 발행조건 및 상환에 미치는 영향을 분석한다. 여기에서 수의상환 채권은 전환 및 비전환 채권을 포함한다. 논문은 전략적 의사결정인 주체인 경영자-주주와 가격순응자인 채권자간의 게임 모형을 채택한다.

기업이 비전환 수의상환 채권을 발행하는 경우, 옵션이 부가되지 않은 표준채권을 발행하는 경우와 같은 균형 파산확률을 가지므로 수의상환 옵션은 재무적 곤경 비용 관련하여 이점을 갖고 있지 않다.

그러나, 재무적 곤경 비용은 수의상환 옵션의 행사 시기에 영향을 미친다. 재무적 곤경 비용이 클수록 수의상환 가격은 상대적으로 점증하는 경향을 가지며, 따라서 기업은 가격 부담을 줄이기 위해 수의상환을 앞당기는 유인을 갖는다. 수의상환은 기업에 재무구조를 조정하는 기회를 제공하므로, 이 결과는 재무적 곤경비용이 클수록 기업은 수의상환 옵션을 행사할 유인이 큼을 시사한다.

기업은 전환옵션부 수의상환 채권을 발행하면 비전환 채권에 비해 조기상환이나 부도 가능성을 낮출 수 있다. 즉, 전환 수의상환 채권을 발행함으로써 차입의 이점을 향유하면서 기대 재무적 곤경 비용을 낮출 수 있다. 이는 전환채권은 기업의 실적이 저조할 경우 전환옵션이 행사되도록 고안된 데에 기인한다.

또한, 이 결과는 정보비대칭 환경에서 우량 기업은 비전환 채권을 발행함으로써 기업가치를 분리신호할 수 있음을 시사한다. 기업은 비전환 채권 발행을 통해 신호비용을 부담할 수 있기 때문이다. 나아가, 기업실적이 저조한 시기에 전환을 유도하는 수의상환 옵션 행사가 빈번한 현상을 설명할 수 있다. 결국, 전환사채에 대한 수의상환 옵션의 행사는 기업가치에 대한 부정적 정보로 이해될 수 있다.

전환채권의 균형 수의상환 가격은 비전환 채권과는 달리 재무적 곤경 비용에 의해 일률적으로 영향을 받지 않는다. 수의상환 옵션의 행사는 주식 전환 및 재무적 곤경 가능성의 소멸을 함께 수반하므로, 복합적 효과를 갖기 때문이다.

색인어: 수의상환채권, 전환채권, 재무적 곤경 비용

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