Korean Management Review, Vol. XIV, No. 1 (Sep. 1984)

Parameter Estimation and Evaluation for the Thurstone Case III and Case V Model

*Seoil Chaiy**

《Contents》

〈國 文 抄 錄〉

社會科學者들은 人間의 選擇過程에 관심을 가지고 많은 연구를 해왔다. Thurstone의 比較判斷模型은 인간의 선택과정을 설명하는 하나의 모델로서 사회과학 분야의 전반에 걸쳐 널리 활용되었다. Thurstone의 모델은 실험대상자들로부터 雙別比較(paired comparison)를 통하여 順位尺度를 얻어서 각각의 個體(object)에 대한 區間尺度를 얻어내는 방법으로 널리 사용되었다. 이 比較判斷模型은 여러가지의 subcase들로 발전되었는데 이제까지 개발된 比較判斷模型 중에서 Case V가 가장 쉽게 尺度値를 구하는 방법이었기 때문에 널리 사용되어 왔다. Case V는 實驗要素(stimuli)가 빛의 밝기 색상등과 같은 심리학적 요소들에 대한 측정을 할 때와 같이 同質的이고 단순한 비교일 때는 효율적인 모델일 수 있으나 우리가 경영학, 특히 마아케팅에서 사용하는 異質的이고 복잡한 사물에 대한 비교를 할 때에는 Case V에서 사용되는 가정이 맞지 않기 때문에 사물들의 서로 다른 특성(특히 각각의 실험 대상자들에게 틀리게 받아 들여 진다는 점)이 標準偏差의 개념으로 나타나게 되는 모델의 Case III 필요성이 있다.

* Korea University

이 논문의 목적은 이제껏 많은 노력에도 불구하고 밝혀지지 않은 Case III 모델의 尺度値를 얻어내는 방법과 그 解의 評價를 하려는 것이다. 解의 추정방법으로는 最小自乘法, maximum likelhood와 minimum chi-square를 이용한 방법이 사용되었으며 이들에 대한 最適解 산출과정은 컴퓨터를 이용한 두가지의 最適解 산출과정이 사용되었다. 즉 Gradient search와 Direct search 방법이 사용되었다.

위의 추정방법과 最適解 산출과정에 관한 조합이 6가지 다른 解法들을 monte carlo simalation을 통하여 평가함으로써 比較判斷模型을 사용하는데 지침이 되도록 하였다. simulation의 결과로는 첫째 sample의 수가 적을 때에는 Case V 모델이 사용되어야 하며 最小自乘法이 maximum likelihood나 mimum chi-square보다 解의 정확도는 낮으나 사용상 큰 영향을 미칠 정도는 아니며, Gradient search 방법을 이용하면 계산시간이 상당히 적게 들기 때문에 실제 사용에 편리하다는 것이다. 마지막으로 사용자가 Case III와 Case V중 어느 모델을 사용되어야 하는 가를 판단하기 위하여 통계적인 방법이 이용되는 maximum likelihood test가 사용되었으며 이 test의 판별력은 양호하였다.

I. Introduction

Economists and psychologists have proposed a number of theories and models that deal with individual decision making. One area of decision making theory deals with how individuals make dominance judgements. Given a single dimension, X, the theory seeks, to expalin how an individual decides if one item, A, has more of X than another item, B. The dimension X may be defined as an attribute or as a utility. Thus, dominance judgements may be concerned with perception, cognition, or perference.

Psychologists have developed various choice axioms to deal with the investigation of decision making. One of the most notable is the theory of choice proposed by Thurstone 1927. Even though Thurstone's model has received a lot of attention during the last 50 years, surprisingly little work has been done in developing estimation procedures for it. The purpose of this paper is to develop and evaluate new estimation procedures for one form of Thurstone's model, that which is commonly known as the Law of Comparative Judgement.

Thurstone's Law of Comparative Judgement

The complete from of the law of comparative judgement may be stated as:

(1) $\mu_i - \mu_j = z_{ij} (\sigma_i^2 + \sigma_j^2 - 2\rho_{ij} \sigma_i \sigma_j)^{1/2}$

where μ_i, μ_j denote the scale (expected) values of stimuli (random variables) i and j.
σ_i, σ_j denote the discriminal dispersions (standard deviations) of stimuli i and j.
p_{ij} is the correlation between the stimuli i and j.
z_{ij} is the normal deviate corresponding to the theoretical proportion of times i is judged to dominate j.

The basic difficulty with the law of comparative judgement is that regardless of the number of stimuli, there are too many unknowns. For example, with n stimuli, there are n scale values, n discriminal dispersions, and $n(n-1)/2$ correlations which are unknown. If the scale value of one stimulus is chosen as an origin its discriminal dispersion as the unit of measurement, $2(n-1)+n(n-1)/2$ unknowns remain. However, paired comparisons give us $n(n-1)/2$ observation equations. Therefore, the number of equations is always $2(n-1)$ less than the number of unknowns In his law of comparative judgement, Thurstone [1927] presented five cases with different degrees of simplifying assumptions. A summary of the five different cases is as follows:

Case I: cemplete form of the law for paired comparison data, which is obtained from a single subject, is used when only a dichotomous judgement is allowed.

Case II: Instead of single observer, a group of observers is used to make judgements.

Case III: The correlation ρ_{ij} is assumed to be zero, 1 i.e., stimuli are independent with each other. With this assumption, equation (1) reduces to:

(2) $\mu_i - \mu_j = z_{ij} (\sigma_i^2 + \sigma_j^2)^{1/2}$

Case IV: The discriminal dispersions are assumed to be small. With this assumption, equation (2) reduced to:

(3) $\mu_i - \mu_j = z_{ij} [1/2(\sigma_i + \sigma_j)]$

which is linear.

Case V: All the discriminal dispersions are assumed to be equal, i.e., all the stimuli have a standard deviation of σ. Then equation (2) becomes:

(4) $\mu_i - \mu_j = z_{ij}\ 2\sigma$

Since the assumed constant discriminal dispersion is the unit of measurement, we can, without loss of generality, set the discriminal dispersion epual to one and obtain:

(5) $\mu_i - \mu_j = 2z_{ij}$

Interest in Thurstone's comparative judgement model has centered around the Case V version. The major reason for this is probably the availability of an explicit Case V estimation procedure with least squares properties [Mosteller, 1951a].

Despite the simplicity of the Case V model, more general forms of the model, in particular the Case III variation, are worthy of further investigation. In those studies that have looked at Case III models, usually in the context of successive interval experiments, or at Case IV models, a sizable range of dispersions is a common finding [Sjöberg, 1965]. Benefits derived from using a more general model, such as Case III, will of course vary with the phenomenon being studied. For some psychophysical stimuli, there may be justifiable theoretical and empirical reasons for staying with a Case V or Casd VI model. For other stimuli, such as those encountered in consumer behavior applications where subjects are asked to compare heterogeneous stimuli of differential uncertainty, the benefits of using a Case III model may be significant. In these situations, the standard deviation associated with a stimulus may often be interpreted as a measure of uncertainty.

Employment of a Case III model can at times result in improved scale value estimates, even where knowledge of the standard deviations has no substantive interpretation. However, estimation of Case III parameters may not lead to better recovery of the scale values, even when data are known to be generated by a Case III process. Uniqueness and estimation problems involved in obtaining a Case III solution may be so severe that one is better off with the simpler Cave V approach to comparative judgments [Suppes and Zinnes, 1963]. Belief that the difficulties encountered with the Case III comparative judgment model outweigh its potential benefits has probably lead to its limited use in psychology and near total absence in applied fields.

The primary purpose of this paper is to explore the merits of using a Case III model. In doing this, the issues of optimization procedure selection and estimation criterion selection are also addressed. A Monte Carlo study is undertaken in which data are generated by Case III processes. The processes differ with respect to the relative magnitude of the standard deviations to the mean values and the number of observations. No distinction is made as to the nature of the sampling unit; sampling errors, though, are assumed to be independent [Bock, 1958]. Estimaticn of Case III parameters is first discussed under two headings, estimation criteria and optimization procedures.

Three statistical estimation criteria and two optimization procedures are proposed. The methodology of the study is then described and results pertaining to (1) the selection of an estimation criterion, (2) the selection of an optimization procedure, and (3) the relative recovery of parameters using Case V and Case III models are reported. The final section addresses the problem of deciding which model to use when the generation process is known to be a Case III process.

II. Estimation Criteria

Thurstone's theory of comparative judgment associates with every stimulus object σ_i a discriminal process of the form $\mu_i + X_i$, where the μ_i are the scale values of the stimuli and the X_i are independently distributed random variables in the Case III situation and independent identically distributed random variables in the Case V situation. When two stimulus objects σ_i and σ_j are presented to a subject, σ_i is selected if the value of the discriminal process for σ_i is greater than the value of the discriminal process for σ_j. In this paper as in the original work [Thurstone, 1927], the discriminal processes are assumed to be normal random variables. The estimation task for the Case III model is to approximate μ_i and $\mathrm{Var}(X_i) = \sigma_i$, while for the Case V model, only the first estimate is required. The comparative judgment model assumes that the estimates are to be derived from the observed frequencies n_{ij} or the proportion of time P_{ij} with which σ_i is chosen over σ_j.

A variety of statistical and non-statistical criteria have been proposed for obtaining estimates of the parameters of the Case III and Cave V models. Three statistical estimation criteria, least squares, maximum likelihood and minimum chi-square, will be evaluated here.

1. Least-squares

The most popular estimation procedure is Mosteller's least squares solution for the Case V model. Mosteller's least squares solution seeks parameter estimates that minimize the quantity

(1) $$Q = \sum_{i<j} (z_{ij} - \hat{z}_j')^2$$

where $$z_{ij} = \Phi^{-1}(p_{ij}),$$

$$\hat{z}_{ij}=x_i-x_j,$$

and where Φ^{-1} is the inverse normal transform of the observed proportions p_{ij} and x_i is the estimate of μ_i. Alternatively, (1) may by expressed in matrix form as

$$Q=(z-Ax)'I(z-Ax) \tag{2}$$

where A, for n stimuli, is an $n(n-1)/2$ by n matrix of coefficients, z is a $n(n-1)/2$ vector with elements z_{ij}, $i<j$ and x is a vector of length n. If the k th row of A, corresponds to the stimulus pair i, λ $(i<j)$, then

$$\begin{aligned} a_{kl} &= 1 \text{ if } l=i \\ &= -1 \text{ if } l=j \\ &= 0 \text{ otherwise.} \end{aligned}$$

The familiar matrix expression for the least squares solution,

$$x=(A'A)^{-1}A'z$$

is the same as Mosteller's solution after adding a constant to permit $\sum x_i=0$. (Since A is of rank $n-1$, a column of A must be dropped and the corresponding value of x_i set equal to 0 to achieve a solution).

A least squares Case III model minimizes the same quantity as (1) except that

$$z_{ij}=(x_i-x_j)/(s_i^2+s_j^2)^{1/2} \tag{3}$$

where s_i^2 is the estimate of σ_i^2. In this situation, the estimate of z_{ij} cannot be expressed as a linear function of the parameter space and an explicit least squares solution procedure is not feasible. Iterative procedures are thus necessary.

Goodness of fit for the least squares model is usually obtained by means of the arc sin transformation suggested by Mosteller [1951b]. The test evaluates the null hypothesis that the model is correct. Thus, acceptance of the null hypothesis may be aided by small samples or poor experimental procedures [Grant, 1962].

2. Maximum Likelihood

For the least squares solution to have minimum variance properties, the expected value of z must depend on a linear combination of the parameter space. This means that

$$z=A\mu+\varepsilon,$$

where ε is a randm vector of zero mean. However, as Bock and Jones [1968] have shown, ε does not have zero mean, but includes a bias factor.

Maximum likelihood estimates may be found by maximizing the likelihood function

$$(4) \qquad L=\prod_{i<j}\binom{N}{n_{ij}}\hat{p}_{ij}^{\,n_{ij}}(1-\hat{p}_{ij})^{N-n_{ij}}$$

where $\hat{p}_{ij}=\Phi(x_i-x_j)$,

$N=n_{ij}+n_{ji}$.

In this expression, Φ is the adf of the normal. An explicit solution of (4) does not exist and iterative methods must be used. Although maximum likelihood estimates are asymptotically efficient, for small sample sizes. they may not be better than least squares.

A maximum likelihood Case III solution may be found by minimizing a function similar to (4) except that

$$\hat{p}_{ij}=\Phi((x_i-x_j)/(s_i^2+s_j^2)^{1/2}).$$

An advantage of the maximum likelihood approach is that the likelihood ratio can be used to compare the Case V and Case III solutions. However, the likelihood ratio test, like the maximum likelihood estimates, rests on asymptotic theory.

3. Minimum Chi-Square

Several minimum chi-square criteria are available [Berksoy, 1956]. Perhaps the most common is the Pearsonian chi-square which minimizes the quantity

$$(5) \qquad Q=\sum_{i<j}n_{ij}/\hat{p}_{ij}-(1-\hat{p}_{ij})(p_{ij}-\hat{p}_{ij})^2.$$

Asymptotically, this criterion is similar to that of the likelihood criterion, and as is the case with the likelihood criterion, a solution must be found by iterative methods.

Another chi-square approach, sometimes called a minimum normit chi-square [Berkson, 1956], minimizes the sum of squared differences between transformations of the proportions; specificially, it minimizes

$$(6) \qquad Q=\sum_{i<j}w_{ij}(z_{ij}-\hat{z}_{ij})^2,$$

where $w_{ij}=(n_{ij}\ \phi^2\ (z_{ij}))/(\hat{p}_{ij}(1-\hat{p}_{ij}))$

and ϕ is the pdf of the normal.

Depending on how $\hat{p}_{ij}$ is defined, this criterion can be used with the Case V or Case III model. The chi-square properties of (6) have been established by Taylor [1953]. Values of x_i and s_i can be found through iterative procedures. If the estimates of the proportions used in w_{ij} are derived from prior analysis, such as a least-squares Cave V analysis, then (6) can be solved explicitly. This can be shown by expressing (6) for the Case V model in matrix from as

(7) $$Q=(z-Ax)'\ W(z-Ax)$$

where W is an $(n(n-1)/2)$ by $(n(n-1)/2)$ diagonal matrix with weights w_{ij} along the diagonal. From (7) it is obvious that an explicit solution does not exist for the Case III model.

Chi-square criteria have a number of advantages over the maximum likelihood methods. One advantage is that they are conceptually and computationally simpler. Also, the Pearsonian expression does not require knowledge of the density function. Chi-square criteria can be used to provide a test of the null hypothesis that the model being evaluated is correct. In addition, for n stimuli, the statistic

(8) $$X_n^2=Q_V=Q_{III}$$

where Q_V is the value of (5) for the Case V model and Q_{III} is the value of (5) for the Case III model can be used with n degrees of freedom to determine if there is a significant advantage in using the Case III model, similarto the likelihood ratio test.

III. Optimization Procedures

Two optimization procedures were used in this study to provide solutions that could not be solved explicity. One was a direct search procedure developed along the lines of the Hooke and Jeeves [1961] method and the second was a gradient search procedure based on a quasi-Newton algorithm [Walsh, 1975].

Direct search procedures are not as elegant as gradient search procedures and they are much slower as well. However, with rough response surfaces, such as might be expected with Case III problems, the direct search procedures should do better than gradient search procedures. Gradient search procedures work best with quadratic functions.

The direct search algorithm used was STEPIT [Chandler, 1969]. The program ZXMIN [IMSL, 1979] was used for the gradient search algorithm.

Numerical optimization procedures can be very sensitive to initial configurations. This is especially true of gradient procedures. For Case V models, the Mosteller solution was always used as the initial configuration. For the Case III models, the only explicit solution for comparative judgments that has been published is the one by Burros and Gibson [1954[. (Explicit Case IV solutions have reported by Thurstone [1932], Gibson [1953] and Burros [1951]). The Burros and Gibson solution is a two stage estimation procedure which first estimates variances and then, given the variance estimates, proceeds to estimate means. This procedure was first used to provide an initial conf iguration for the Case III model. The Burros-Gibson estimates of the variances, however, proved to be particularly poor. Trial of the Case IV solution by Burros [1951] was much moJe satisfactory and was used throughout this study as the initial Case III solution.

IV. Methodology

Three estimation criteria and two optimization procedures have been described in the previous sections. The methodology of this study sought to determine how well those three criteria and two procedures did at recove ring the parameters of two Thurstone models when the data were generated by Case III processes.

1. Recovery

Selection of a recovery measure depends upon the measurement properties of the estimates. The means (the scale values) of the Thurstone Case V model are unique up to a linear transformation and standard deviations up to a positive proportionality transformation. Uniqueness properties of the Thurstone Case III model are more complicated. Suppes and Zinnes [1963] have shown that scale values for the Case III model have something less than interval scale properties. They demonstrated this in a counter example involving three stimuli. We suspect that the departure from linearity diminishes as the number of stimuli increases. We will therefore assume uniqueness properties of the parameters in the Case V and Case III models to be the same.

The product moment coefficient of determination R^2 was chosen to measure the recovery of the scale values. The coefficient R^2 is an intuitively meaningful measure and it is unique up to a linear transformation. For the recovery of the standard deviations, a measure was desired that was similar to the product moment measure but which was in variant over multiplicative transformations but not all linear transformations. One such measure is

$$R_U{}^2 = R_SS_U / TSS_U = TSS_U - ESS_U / TSS_U,$$

where RSS_U is the regression sum of squares unadjusted for the mean, TSS_U is the total sum of squares unadjusted for the mean, and ESS_U is the error sum of squares unadjusted for the mean.

The R_U^2 coeffient is probably the most popular measure of comparison for zero intercept models but it has the unfortunate property of frequenly being higher than the value of R^2, when one would usually like a more constrained measure to be lower. A number of alternatives to R_U^2 have been proposed [Aigner, 1971] but none are ideal. The most common fault of alternatives to R_U^2 is that they are unbounced on one end. Both Aigner and Theil [1971] favor a relative conservative measure R_A^2 which is like R_U^2 except that the total sum of squares measure is adjusted for the mean. R_A^2 is therefore always less than R^2. It does, unfortunately, have an undetermined lower bound, with negative values being quite common in cases of poor fit. Note, though, that negative values of R_A^2 do not indicate a negative relationship in the two variate situation. Also, $R_A^2(x \cdot y)$ in general does not equal $R_A^2(y \cdot x)$. Since comparison of the recovery of the μ and σ parameters was desired, R_A^2 was selected as the measure of recovery for σ and R^2 was selected as the measure of recovery for μ.

2. Data

Ten stimuli were used in the analyses. If the case III model can be shown to do well for ten stimuli, then we may suppose that the uuiqueness difficulties cited by Suppes and Zinnes for three stimuli arelikely to be unimportant in most a pplications.

Scale values μ_i were randomly selected from a unfiorm distribution between -0.5 and 0.5. Error parameters σ_i were selected from a uniform distribution between 0.0 and an upper bound σ_U. Four levels of σ_U.

Four levels of σ_U were used, 0.25, 0.5, 1.0 and 2.0. The ratio of σ_i to σ_j was constant for all four levels of σ_U.

The number of observations N was also varied. Seven levels were used: 10, 20, 40, 80, 160, 320 and 640. For each observation, 45 pairs of values were drawn, each value coming from a normal distribution with parameters μ_i and σ_i. If the value drawn for stimulus i was greater than the value drawn for stimulus j, the frequency with which i dominated j was increased by one. For each combination of error and observation levels, ten replications were evaluated.

3. Monte Carlo Procedures

All of the algorithms used to specify density functionsand their inverses came from the International Mathematical and Statical Library [IMSL, 1979]. The basic random number generator used by IMSL is a multiplicative congruential generator. To decrease the likeli hood of irregularities, numbers were drawn in lots of 500 and then shuffled [Chambers, 1977]. The initial seed for the random number generator was drawn from a random number table. A copy of the program used is available on request from the senior author.

4. Analysis

Least squares, maximum likelihood and minimum chi-square criteria were used to estimate Case V and Case III parameters of each of the 280 sets of data. Both the Pearsonian chi-square and the normit chi-square were initially used for the Case V model. The normit chi-square procedure was of course quicker and also had better recovery. For Case III analyses, the normit criterion (used iteratively) did no better than the simpler Pearsonian chi-square. Thus, the normit chi-square was used for all Case V analyses while the Pearsonian chi-square was used for Case III.

Interative procedures were requried for all three Case III estimates and for the maximum likelihood Case V estimate. The Burros procedure was used to provide the initial solution's for all Case III analyses and the least squares [Mosteller] procedure provided the initial solution for the Case V maximum likelihood analyses.

V. Results

Results of the Monte Carlo analyses are presented in terms of the three decisions addressed by this paper; selection of an optimization procedure, estimation criterion, and Case V or Case III model.

1. Optimization Procedures

A direct search and gradient search procedure were evaluated. It was expected that the direct search should do better than the gradient search when encountering rough response surfaces. In terms of the conditions evaluated here, rough response surfacesare most likely when there is a high degree of error, a low number of observations, and a Case III model.

The Case V analysis of 280 data sets (4 levels of error, 7 levels of observations, 10 replications, maximum likelihood criterion) using both optimization procedures resulted in the same mean level of recovery (R^2) for the means, 0.85. While each individual data set did not always have the same level of recovery using direct search and gradient search, there were no systematic differences by level of error or observation. It is thus concluded that for Case V models, the more efficient gradient search procedure is better. To simplify further analyses, only gradient search procedures were used for the maximum likelihood estimates of the Case V model in the rest of this paper.

With the Case III model, iterative optimization procedures were used for all there. estimation criteria. Recovery of both means and standard deviations was evaluated. Average recovery of the means was again identical, 0.84. The direct search procedure did, though, show a slight improvement over the gradient search procedure when the criterion was one of standard deviation recovery (R_A^2), −.12, vs. −.19.

A fixed effects ANOVA for the Case III model, using recovery of the standard deviations as the dependent variable, was conducted using four factors: optimization procedures(2 levels), estimation criteria (3 levels), error (4 levels) and observations (7 levels). No significant main effects or interactions with respect to the optimization procedure factor were found. Since the direct search procedure was marginally better, it was decided to use only the direct search procedure for Case III models. It is clear, though, that a good gradient search procedure will do almost as well as the less efficient but more through direct search procedures.

2. Estimation Criteria

Two analyses of variance for the Case V model were conducted involving three factors: estimation criteria (3levels), error (4 levels) and observations (7 levels). The first analysis used R^2 as the recovery measure and the second used Fisher's Z transformation $Z=1/2\ \ln[(1+R)/(1-R)]$ to compensate for the skewness of the correlation measures. Results are presented in Table 1.

Both analyses attribute the greatest explanation to the levels of observation, error and their interaction. Use of the Z criterion makes the criteria factor and the criteria-error interaction significant but their relative importance is still minor. Analysis of the

Table 1. **Analysis of Variance Summary: Recovery of Means for the Case V Model**

Source of Variation	df	R^2			Z		
		Mean Square	F	Probability	Mean Square	F	Probability
Criteria (C)	2	0.0	0.1	N.S.	2.7	28.1	<.001
Error Level (E)	3	3.2	269.2	<.001	57.0	590.2	<.001
Observation Level (O)	6	1.5	127.4	<.001	19.3	199.7	<.001
CE	6	0.0	0.3	N.S.	1.1	11.2	<.001
CO	12	0.0	0.1	N.S.	0.2	1.65	N.S.
EO	18	0.3	25.1	<.001	0.8	8.69	<.001
CEO	36	0.0	.0	N.S.	0.7	.74	N.S.
Residual	756	.01			756	.10	

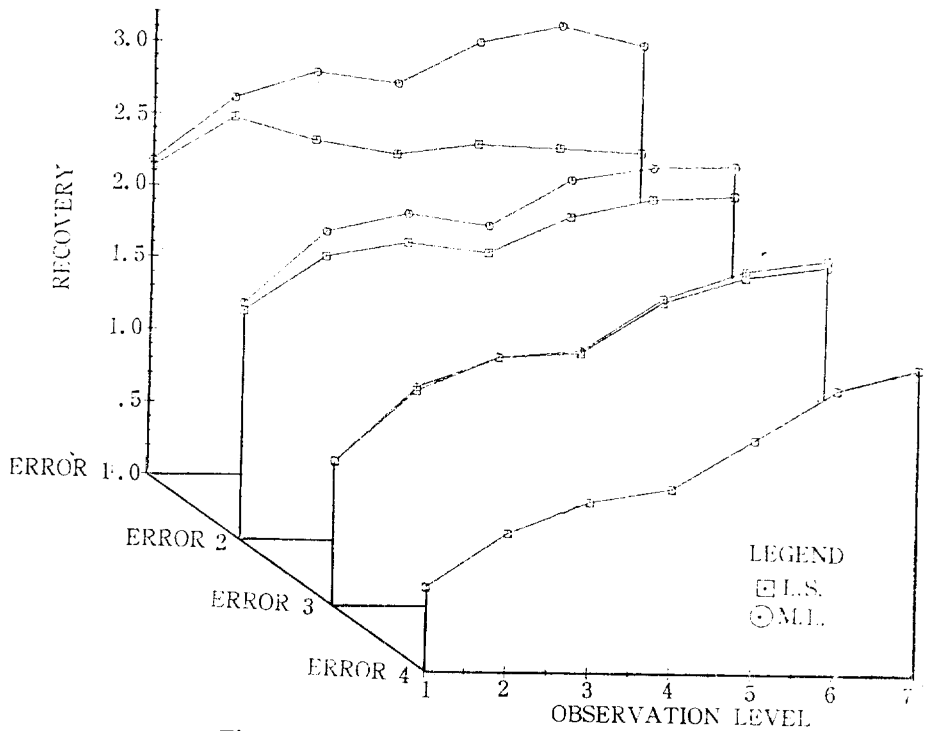

Figure 1. Recoverd of Case V scale values.

means showed that the maximum likelihood criterion was best and least squares the worst. The chi-square criterion was very close to the maxmum likelihood criterion in all situiations. A plot of the means of the Z criterion for the least squares and maximum likelihood estimators is given in Figure 1. It will be noticed that the maximum likelihood criterion never does worse than the least squares criterion, though at error level four they are indistinhuishable. As expected, recovery generally improves as the error level decreases and as the number of observations increases. The exception to this is least squares at error level 1. In this situation, the standard deviations were in a narrow range close to zero. The data, generated by the Case III model, were not handled as well by the linear model. Relative performance of the asymptotically efficient maximum likelihood procedure was greatest at high observation levels. Though not evident here, it is likely that least squares could do better than maximum likelihood at high error levels with few observations.

Three analyses were conducted for the Case III model. The designs were similar to that for the Case V model, the dependent variables for the recovery of the means

Table 2. Analysis of Variance Summary: Recovery of Means for the Case III Model

Source(of Variation	df	R^2			Z		
		Mean Square	F	Probability	Mean Square	F	Probability
Criteria (C)	2	0.0	1.2	N.S.	3.0	30.1	<.001
Error Level (E)	3	4.5	346.9	<.001	97.4	977.4	<.001
Observation Level (O)	6	2.7	211.7	<.001	65.5	657.0	<.001
CE	6	0.0	0.9	N.S.	2.6	25.6	<.001
CO	12	0.0	0.1	N.S.	0.0	0.3	N.S.
EO	18	0.4	28.4	<.001	0.2	2.0	<.001
CEO	36	0.0	0.1	N.S.	0.0	0.3	N.S.
Residual	756	0.0			0.1		

Table 3. Analysis of Variance Summary: Recovery of Standard Deviations

Source of Variation	df	Mean Square	F	Probability
Criteria (C)	2	7.7	9.2	<.001
Error Level (E)	3	63.6	76.3	<.001
Observation Level (O)	6	134.4	161.2	<.001
CE	6	5.2	6.2	<.001
CO	12	5.2	6.2	<.001
EO	18	5.1	6.1	<.001
CEO	36	0.3	0.4	N.S.
Residual	756	0.8		

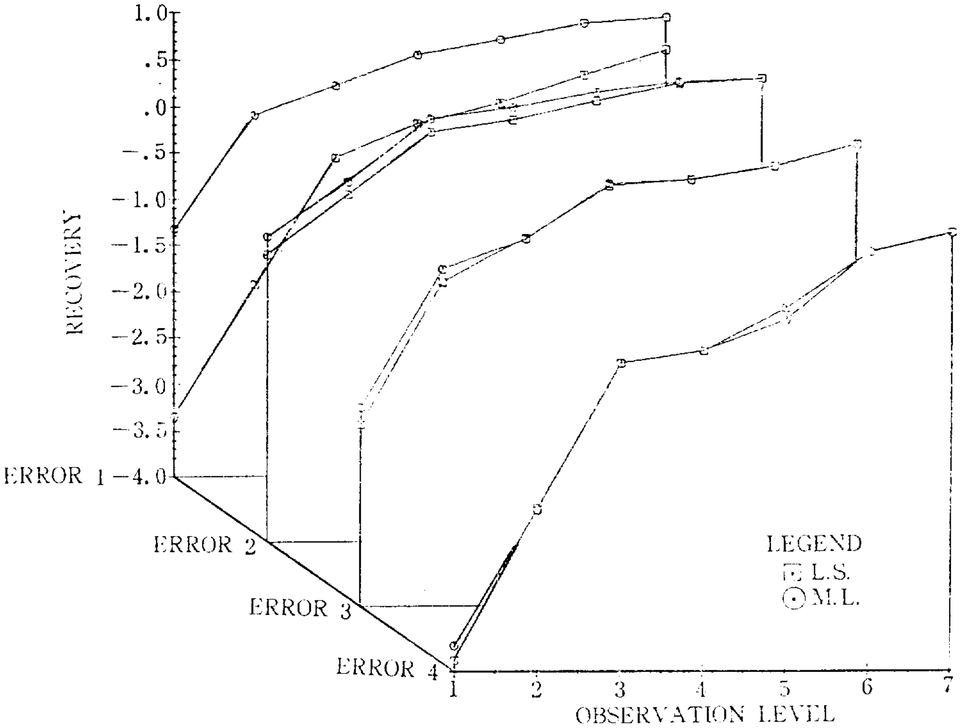

Figure 2. Recovery of standard deviations.

being R^2 and Z and for recovery of the standard deviation, $R_A{}^2$. The overall ranking of the three estimation criteria for the Case III model was the same as that for the Case V model with respect to the means. For the standard deviations, the minimum chi-square criterion was insignificantly ahead of the maximum likelihood criterion. Results for the ANOVA of the means is given in Table 2. The pattern is similar to that with Case V. Table 3 reports on the recovery of the standard deviations. The significance of the criterion main effect and interactions that are indicated in Table 3 is illustrated in the plot of recovery values for the standard deviations in Figure 2. For error levels 3 and 4, the three criteria do equally well. (The chi-square criterion is omitted for clarity and consistency with the other figures. It is very similar to the maximum likelihood criterion in performance). As with the recovery of the means, the maximum likelihood criterion has the greatest relative advantage over least squares at the low error levels. The criterion-observation interaction is evident in the diminishing difference between the two estimation criteria as the number of observations increases. The nature of the interaction was surpring, as our prior expectation was that the

Table 4. Frequencies of R_A^2 and R^2 for the Maximum Likelihood Estimates of the Standard Deviations

		R_A^2			
		<0.00	(0.00~0.5)	(0.50~0.75)	>0.75
	<0.25	27	0	0	0
R^2	(0.25~0.50)	44	6	0	0
	(0.50~0.75)	25	39	14	0
	>0.75	2	8	23	89

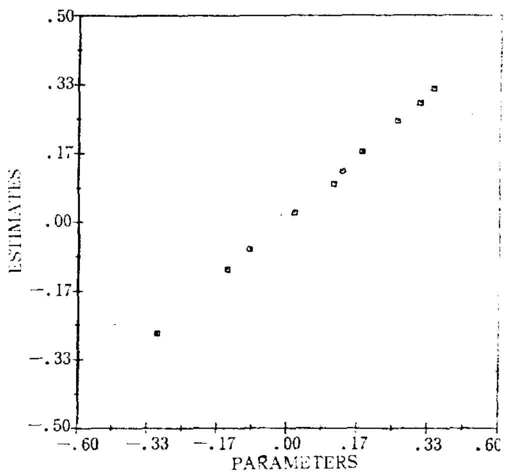

Figure 3. Mean estimates of scale values.

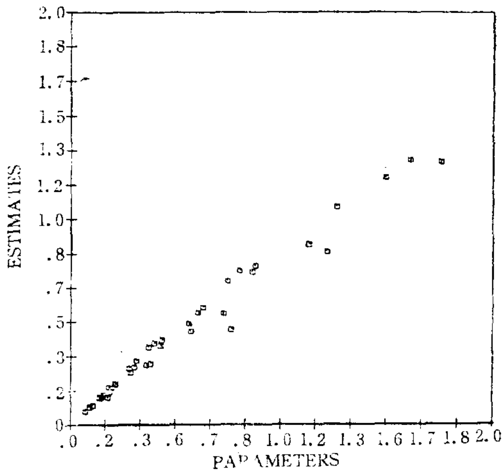

Figure 4. Mean estimates of standard deviations.

relative difference would increase as the number of observations increased.

One of the striking features of Figure 2 is the generally low value of the recovery criteria. At no time is the recovery for the standard deviations higher than the recovery for comparable means. Values of R_A^2 exceed 0.9 only for situations involving over 320 observations, and then only for low error levels. Since the R_A^2 criterion is not as well known as the R^2 criterion for linear models, a cross tabulation of the two statistics for the 280 maximum likelihood estimates of the standard deviations is given in Table 4. The conservative nature of the R_A^2 statistic is evident from the table. On two occasions, data which had a negative R_A^2 relationship had corresponding values of R^2 greater than 0.75. The frequent occurrence of negative R_A^2 raises the question of bias in the estimates. Plots of the parameters and maximum likelihood estimates for the means and standard deviations show a modest downward bias for standard deviations

Table 5. **Analysis of Variance Summary: Case III and Case V Recovery of Means**

Source of Variation	df	R^2			Z		
		Mean Square	F	Probability	Mean Square	F	Probability
Model (M)	1	.1	5.5	<. 50	2.3	23.3	<.001
Error Level (E)	3	2.7	219.5	<.001	65.3	655.3	<.001
Observation Level (O)	6	1.3	108.3	<.001	2.3	23.3	<.001
ME	3	0.0	2.0	N.S.	1.1	11.7	<.001
MO	6	0.0	2.1	<.050	2.5	24.6	<.001
EO	18	0.2	18.8	<.001	0.1	1.2	N.S.
MEO	18	0.0	0.3	N.S.	0.1	1.4	N.S.
Residual	504	0.0				0.1	

Figure 4) and almost no bias for estimates of the mean (Figure 3). Corresponding plots for the least squares estimates (not shown) have the same degree of bias. The estimates of the means are averaged over 280 cases and the 40 standard deviation essimates (10 for each error level) are averaged over 70 cases. (Since the numerator and denominator of equation (3) can both be multiplied by any arbitrary constant, it was necessary to standardize the mean and standard deviation estimates. This was done by finding the factor that equated the standard deviations of the scale parameters and the estimates of the means for each case and applying this factor to both the mean and standard deviation estimates).

In almost all situations, recovery using least squares was inferior to recovery using maximum likelihood and minimum chi-square. For this reason least squares estimates will be omitted in the remainder of the paper.

3. Model Choice

All of the data used in this study were generated by a Case III model. One might expect that a Case III estimation procedure would always do best at recovering the means (scale values) but such is not the case. An ANOVA for the maximum likelihood solution using recovery of the means (R^2 and Z) as the dependent variable was conducted involving three factors: model (2 levels), error (4 levels) and observations (7 levels). Results are in Table 5.

The model main effect and the model-observation interaction was significant with

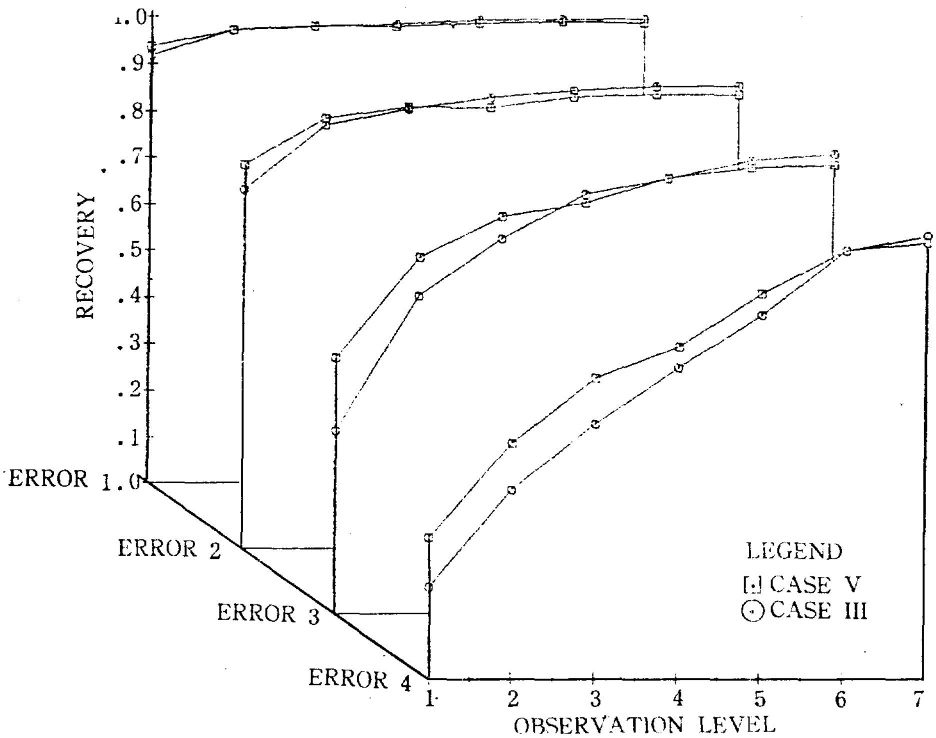

Figure 5. Recovery of scale values-Case IIIand Case V.

both R^2 and Z. As before, results were more significant with Z. Interestingly, the analysis with Z gave much higher significance to the model-error interaction than to the error-observation interaction, reversing the findings of the R^2 analysis. In either case, it is obvious that the selection of the Case V or Case III model does make a difference. The same conclusion can be drawn from analyses using the minimum chi-square criterion (not shown).

Mean recoveries (R^2) of the maximum likelihood scale value estimates for the Case V and Case III models are plotted in Figure 5. The Case III model's relative advantage comes with increasing numbers of subjects. At lower error levels, the advantage is realized earlier, with fewer observations, than at high error levels. Apparently, the roughness of the Case III response surface at high levels of error or low levels of obse-rvations is severe enough so that better estimates cannot be achieved with the Case III model. The greater degrees of freedom available with the Case V model leads to superior recovery under these conditions.

Given a set of data which may be thought to be generated by a Case III process,

Table 6. Acceptance of Case III and Case V Models for Three Likelihood Ratio Tests

A	Model with Best Recovery	Percentage Accepting III	Percentage Accepting Case V	Number
0.900	Case III	84	16	153
	Case V	26	74	127
0.950	Case III	82	18	153
	Case V	22	78	127
0.990	Case III	75	25	153
	Case V	17	83	127
0.995	Case III	73	27	153
	Case V	17	83	127

the experimenter may still opt for a Case V analysis in those cases where there is no interest in obtaining estimates of the standard deviations. Several suggestions for deciding on which model should be used have been offered. Thurstone [1972] suggested that when two cases were of interest, both should be run and the one which resulted in the best goodness of fit should be chosen. If this procedure is followed, however, the more general model with the greater number of parameters will always be chosen, even though it may have poorer recovery. (To illustrate, the average value of the Pearsonian chi-square estimation criterion when using the Case V model was 71.0 while for the Case III model it was 10.1). A more sophisticated procedure would be to do a statistical test of the null hypotheses that the Case V and Case III models are correct and select that has the least amount of evidence against it. Unfortunately, like the procedure suggested by Thurstone, this procedure will also strongly favor the more general Case III model and it is not capable of distinguishing those cases where the Case V model yields better recovery. With the minimum chi-square criterion for example, evaluation of the α levels for the Case III and Case V models resulted in the selection of the Case V model only sixteen times and for four of these, the Case III model recovered the scale parameters more accurately. Yet, in actually, the Case V model recovered the means better than the Case III model on 118 (42 percent) occasions.

A much more promising approach would be to use a likelihood ratio test, setting strict levels for rejection of the specific model in favor of the more general. model With minimum chi-square estimates, the analagous statistic in equation (8) could be used. Results are presented in Table 6 for using the likelihood ratio test with critical regions

Table 7. **Means and Standard Deviations of Error Range Estimates**

Parameters[1]	Means of Estimates	Standard Deviations of Estimates
0.34	0.30	0.08
0.67	0.59	0.16
1.34	1.25	0.43
2.68	2.80	1.86

[1]Parameters do not equal the boundary limits of equation (9) (0.25, 0.5, 1.0, 2.0) but reflect instead the parameter values drawn for μ_i and σ_i.

corresponding to α levels of .90, .95, .99, and .995. From Table 6, it is apparent that one would have about an 80 percent probability of correctly picking a Case V or a Case III model with an α value criter on of .95.

Another alternative would be to derive a function, based upon the number of subjects and the estimated level of error, for discriminating between those situations where the Case V model has the better recovery and those situations where the Case III model has the better recovery. The success of this procedure would depend upon the degree to which the level of error can be estimated from the data. For the 280 cases used in this analysis, the degree of error was estimated in the same way that it was generated, namely,

(9) $$E=(s_U)/(x_U-x_L)$$

where U and L signify the lower bounds respectively. The resulting mean values in Table 7 (averaged over observation levels) are quite good. Estimates of E could, of course, be improved by calculating a bias correction factor for the estimates of the standard deviations but, even so, the high standard deviations of the E statistic would remain. An initial evaluation of this approach using half of the data set to construct a discriminant equation and then evaluating the resulting equation with the remaining data showed that showed that it performed as well as the likelihood approach with 88 percent of the situations where the Case III model was superior and 75 percent of the situations where the Case V model was superior correctly classified. The relative awkwardness of this alternative, though, leads us to prefer the likelihood ratio test procedure.

VI. Summary

The Monte Carlo studies reported here examined the ability of least squares, maximum likelihood and minimum chi-square methods to recover the parameters of Thurstone's comparative judgment model. Direct search and gradient search procedures were employed. Data were generated by a Case III model under varying degrees of error and numbers of observations. The data were analyzed by both Case V and Case III methods.

It is shown that if one is interested only in recovering means and is limited to small sample analysis, then the traditional least squares Case V analysis, though marginally inferior to maximum likelihood and minium chi-square methods, is probably acceptable. In other situations, it would appear that there are few advantages to using least squares Case V analyses. Case III analyses frequently recover means better than Case V. In addition, they provide standard deviation estimates which, though slightly biased, are reasonably precise. Maximum likelihood and minimum chi-square estimates are almost always superior to least squares, for both Case V and Case III. In addition, they admit a simple yet powerful test for choosing between the Case V and Case III models. No significant difference was found between the direct search and gradient search procedures. Use of efficient gradient search procedures should make Case III analyses reasonable for most applications.

References

1. Aigner, D.J., *Basic econometrics.* Englewood Cliffs, N.J.: Prentice Hall, 1971.
2. Berkson, J., "Estimation by least squaresand maximum likelihood." In J. Neyman (Ed.), *Proceedings of the Third Berkeley Symposium cn Mathematical Statistics and Probability,* **1.** Berkeley: University of Cali fornia Press, 1956, 1-11.
3. Bock, rnD., "Remarks on the test of significance for the method of paired comparisons." *Psychometrika,* 1958, **23,** 323-334.
4. Bock, R.D. & Jones L.V., *The measurement and prediction of judgment and choice.* San Francisco: Holden Day, 1968.
5. Burros, R.H., "The application of the method of paired comparisons to the study of reaction potential." *Psychological Review,* 1951, **58,** 60-66.
6. Burros, R.H. & Gibson, W.A., "A solution for case III of the law of comparative judgment." *Psychometrika,* 1954, **19,** 57-64.
7. Chambers, J.M., *Computational methods for data analysis.* New York: Wiley, 1977.

8. Chandler, J.P., "STEPIT-Finds local minima of a smooth function of several parameters." *Behavioral Science*, 1969, **14**, 81-82.

9. Gibson, W.A., "A least-squares solution for case IV of the law of comparativerjudgment." *Psychometrika*, 1953, **18**, 15-21.

10. Grant, D.A., "Testing the null hypothesis and the strategy and tactics of investigating theoretical models." *Psychological Review*, 1962, **69**, 54-6.

11. Hooke, R. & Jeeves, T.A., "Direct search" solution of numerical and statistical problems. *Journal of the Association for Computing Machinery*, 1961, **8**, 212-229.

12. *IMSL Library*, *Edition* 7. Houston: IMSL, 1979. 13. Mosteller, F., "Remarks on the method of paired comparisons: I. The least-squares solution assuming equal standard deviations and equal correlations." *Psychometrika*, 1951a, 3-9.

13. Mosteller, F., "Remarks on the method of paired comparisons: III. A test of significance for paired comparisons when equal standard deviations and-equal correlations." *Psychometrika*, 1951a, **16**, 3-9.

14. Mosteller, F., "Remarks on the method of paired comparisons: III. A test of significance for paired comparisons when equal standard deviations and equal correlations are assumed." *Psychometrika*, 1951b, **16**, 207-218.

15. Siöberg, L., "A study of four methods-for scaling paired comparisons data." *Scandinavian Journal of Psychology*, 1965, **6**, 173-185.

16. Suppes. P. & Zinnes, J.L., "Basic measurement theory." In R.D. Luce, R.R. Busch, E. Galanter(Eds.), *Handbook of Mathematical Psychology*, *I*. New York: Wiley, 1963, 1-76.

17. Taylor, W.F., "Distance functions and regular best asymptotically normal estimates„ *Annals of Mathematical Statistics*, 1953, 85-92.

18. Thurstone, L.L., "Stimulus dispersions in the method of constant stimuli." *Journal of Experimental Psychology*, 1932, **15**, 284-297.

19. Thurstone, L.L., "A law of comparative judgment." *Psychological Review*, 1927, **30**, 273-286.

20. Walsh, G.R., *Methods of optimization*. New York: Wiley, 1975.

〈부록 1〉 회전以前의 要素行列*

	要素 1	要素 2	要素 3	要素 4	要素 5	要素 6	要素 7	要素 8
X_{60}	0.47498	−0.24045	0.01024	0.03065	−0.19288	0.11887	−0.22967	−0.03744
X_{140}	0.43864	−0.34568	0.06219	0.04049	0.02461	−0.09387	0.02731	0.09330
X_{122}	0.39944	−0.24279	0.27205	−0.14730	0.06644	−0.04103	−0.10855	−0.07879
X_{69}	0.39338	0.07746	0.05478	0.01791	−0.09379	−0.02019	−0.10679	−0.06809
X_{115}	0.38830	−0.18299	0.17620	−0.24043	−0.07636	−0.12333	−0.02289	−0.09333
X_{121}	0.38781	−0.24174	0.22559	−0.20285	0.07635	0.01602	−0.04784	−0.10247
X_{168}	0.38250	−0.00009	−0.14206	−0.21434	−0.16715	0.05459	−0.11021	0.09910
X_{111}	0.37653	−0.19851	0.34622	−0.08332	0.05366	−0.02550	−0.04155	−0.18724
X_{63}	0.37582	0.06565	0.15760	−0.22011	0.07219	−0.11071	−0.13854	−0.02644
X_{133}	0.36935	0.05903	−0.05141	−0.02678	−0.06139	0.05894	−0.00731	−0.07649
X_{126}	0.36878	−0.22628	0.11118	−0.00633	−0.07577	0.02023	−0.03851	0.06826
X_{139}	0.35886	−0.12123	0.13670	−0.17319	−0.12437	0.06162	0.00046	0.14800
X_{114}	0.35638	−0.17415	0.26997	−0.04625	0.04137	−0.01137	0.04603	−0.12590
X_{61}	0.35533	−0.24464	0.05453	0.12838	−0.14010	0.04572	−0.06353	−0.03982
X_{56}	0.35115	−0.20618	0.08594	−0.12202	−0.05038	0.09704	−0.07374	0.04357
X_{73}	0.35058	−0.05677	0.21719	0.12928	0.07898	−0.26351	0.00685	0.02548
X_{77}	0.34986	0.10890	−0.05979	−0.26752	−0.13101	−0.04540	−0.13997	0.10328
X_{113}	0.34904	−0.20599	0.16445	−0.01545	−0.06710	−0.06168	−0.13267	−0.13977
X_{152}	0.34822	0.28785	0.09216	0.17050	−0.20565	0.14027	0.23904	−0.07980
X_{41}	0.34067	−0.09861	0.24825	−0.14948	−0.23180	0.05038	−0.17213	0.06415
X_{138}	0.32740	−0.05749	−0.00554	−0.06186	−0.03400	0.06454	0.01618	−0.04055
X_{70}	0.32703	−0.20013	0.09140	0.04591	−0.13457	0.17615	−0.11168	−0.00419
X_{57}	0.32569	−0.01835	0.07799	−0.16929	0.02397	−0.02857	−0.04983	−0.01650
X_{195}	0.32081	−0.12077	0.08901	0.22143	0.18442	0.04813	0.04308	−0.11895
X_{32}	0.31413	−0.01187	−0.17916	−0.23268	−0.03183	0.10347	−0.05319	0.03378
X_{17}	0.30944	0.20519	−0.09755	0.03891	−0.28471	−0.10846	−0.02259	−0.00253
X_{95}	0.33524	0.05562	−0.18624	−0.06738	−0.14198	0.05170	−0.08446	−0.05237
X_{85}	0.29985	0.06590	0.11213	0.06041	−0.05182	−0.10144	−0.16956	0.03733
X_{53}	0.29082	−0.15859	0.09024	−0.11862	0.03610	−0.07664	0.01653	−0.03723
X_{86}	0.29010	0.06090	0.19395	0.05207	0.18220	0.19556	−0.00519	−0.15471
X_{186}	0.28843	−0.05891	−0.19204	0.07752	0.12212	−0.00843	0.04732	0.04965
X_{8}	0.28281	0.20886	−0.27934	−0.00902	−0.14883	0.00078	0.04217	0.01476
X_{82}	0.28040	0.18741	−0.10560	−0.05723	−0.10263	0.01553	−0.16198	0.08952
X_{74}	0.27786	0.15215	0.15675	−0.24320	−0.11376	−0.05278	−0.10593	−0.03591
X_{63}	0.27466	−0.01189	−0.01964	−0.00840	−0.06494	0.13031	−0.18293	0.08242
X_{2}	0.26878	0.06712	0.08309	−0.02974	0.24511	−0.12775	0.09009	−0.09001
X_{55}	0.26706	0.08265	0.12787	−0.19747	−0.02989	0.12683	−0.15419	−0.09748
X_{159}	0.26197	0.00898	−0.17970	0.25016	0.12729	0.03498	−0.02012	−0.04627
X_{44}	0.26157	0.21849	−0.04798	0.25998	−0.11184	−0.03362	−0.21458	0.07432
X_{203}	0.25798	0.00340	−0.23851	−0.13712	0.06253	0.02874	0.16854	−0.03284
X_{65}	0.25646	0.10289		−0.22917	−0.03358	0.06451	−0.01022	0.05454
X_{131}	0.24600	−0.23972	−0.05571	−0.06085	−0.03931	−0.05390	0.13767	0.08352
X_{119}	0.23555	−0.12412	−0.14380	0.12117	0.01407	−0.10104	0.10028	0.10385
X_{148}	0.23100	−0.16749	−0.00315	0.01935	−0.07417	0.02816	0.01738	0.12679
X_{98}	0.22625	0.12654	0.02293	−0.15613	−0.13314	−0.00078	−0.05851	0.03320
X_{90}	0.22322	0.16919	−0.04337	0.10627	0.16111	0.20124	−0.05703	−0.03727
X_{68}	0.21552	0.07138	0.05360	0.09048	−0.09844	−0.03448	−0.00763	0.14005
X_{84}	0.20478	−0.02313	0.13306	0.09822	0.15625	−0.16062	−0.03972	0.01584
X_{12}	0.20460	−0.06369	0.00486	0.00152	−0.05732	−0.02487	0.03319	−0.01950
X_{34}	0.20151	−0.06063	−0.09761	−0.01285	−0.05483	0.15944	0.06456	0.09448
X_{134}	0.18485	0.02855	0.06159	−0.02803	0.06214	0.14244	0.11146	0.05325

* 紙面관계 및 편의상 要素行列表의 一部만 수록하였음.

〈부록 2〉 회전後의 要素行列*

	要素 1	要素 2	要素 3	要素 4	要素 5	要素 6	要素 7	要素 8
X_{112}	0.58057	0.10957	−0.08699	0.01290	0.00041	0.00752	−0.08334	0.08247
X_{118}	0.50926	0.05352	−0.07080	−0.01905	−0.00077	0.01041	−0.04093	0.03747
X_{140}	0.50359	0.16173	−0.10208	0.08757	0.02714	0.12632	0.06407	0.02040
X_{130}	0.49249	0.04565	−0.03919	0.03364	−0.05346	−0.01246	−0.00114	−0.11864
X_{60}	0.46705	0.01528	0.01795	0.08158	0.28366	0.22327	−0.13317	0.05245
X_{59}	0.46645	0.01602	−0.10052	−0.02055	0.26733	0.02340	−0.08233	−0.06086
X_{100}	0.44374	0.00189	0.02165	0.06547	0.12037	−0.05011	−0.09509	−0.10060
X_{111}	0.43488	−0.05706	0.02759	−0.03248	−0.01592	0.24260	0.10215	0.05061
X_{145}	0.43155	0.10225	0.04503	−0.07612	0.09396	−0.00382	−0.03422	−0.03580
X_{104}	0.42552	0.06053	−0.02535	−0.04996	0.02813	0.02615	−0.03521	−0.07319
X_{114}	0.41983	0.00739	0.07122	−0.06758	−0.02518	0.12664	0.10499	0.11945
X_{126}	0.41535	0.01303	−0.03679	0.10155	0.09053	0.19471	0.04361	0.00606
X_{122}	0.41339	0.01759	−0.05807	−0.01497	−0.01337	0.30779	0.08307	0.07145
X_{61}	0.40852	0.02882	0.07754	0.02435	0.04663	0.17018	−0.11952	0.01589
X_{105}	0.40368	0.04344	0.03976	−0.04114	0.01115	0.01811	0.00473	−0.08398
X_{121}	0.39517	0.03418	−0.02826	−0.11379	0.01612	0.31376	0.08624	0.04251
X_{113}	0.39480	0.01759	−0.01919	0.10148	0.04583	0.08254	−0.03076	0.06643
X_{91}	0.39235	0.00124	0.04804	0.01338	−0.05820	0.09007	0.08103	−0.10338
X_{67}	0.39152	−0.01535	−0.06123	0.08197	−0.04822	−0.06481	0.01444	−0.01353
X_{70}	0.38861	−0.06877	0.03830	0.01378	0.09863	0.19469	−0.06157	0.00217
X_{167}	0.38301	0.18576	−0.06288	−0.02242	0.26554	−0.07694	−0.22157	−0.00539
X_{66}	0.37410	−0.13131	0.05019	0.09894	−0.13830	0.02950	0.01622	−0.05596
X_{143}	0.37162	0.02006	−0.09324	0.02453	−0.10051	0.05667	0.16422	0.05509
X_{201}	0.36713	0.08745	0.09396	−0.01680	0.02209	−0.01229	−0.05656	−0.06552
X_{142}	0.35701	0.13792	0.04924	0.02892	0.00022	−0.02774	−0.09890	−0.06640
X_{56}	0.35604	0.00396	−0.03170	−0.04126	0.18419	0.30353	0.04697	0.00138
X_{106}	0.34624	0.03688	0.09190	−0.00186	−0.08968	−0.01146	−0.02458	−0.05807
X_{115}	0.34035	0.03474	0.00728	−0.05341	0.05575	0.31638	0.00977	0.13523
X_{195}	0.33146	0.17568	0.14599	0.09231	−0.10899	0.03972	0.01450	−0.01831
X_{53}	0.31239	0.05951	−0.10714	0.01357	0.08081	0.22252	0.09659	0.06123
X_{174}	0.30645	0.13606	−0.05956	−0.12955	−0.07202	−0.05370	−0.06953	−0.01088
X_{73}	0.30211	0.09493	0.00082	0.28322	−0.02807	0.08508	0.18766	0.09602
X_{139}	0.29898	−0.01557	0.06046	0.03799	0.23881	0.11367	0.21994	−0.02086
X_{131}	0.29890	0.15186	−0.02650	−0.05383	0.15481	−0.06185	0.03978	0.02612
X_{198}	0.29201	0.03017	0.11383	−0.00399	−0.02105	−0.06578	0.02391	0.04178
X_{149}	0.27904	0.06137	−0.00606	−0.07662	−0.15188	−0.15105	0.10239	−0.02259
X_{165}	0.27548	0.12020	0.00869	−0.15502	0.02035	0.01055	−0.05688	−0.02354
X_{148}	0.26601	0.07072	0.01251	0.10155	0.10024	0.00610	−0.00855	−0.04006
X_{36}	0.25611	0.16084	0.01516	0.19845	−0.06119	−0.03424	−0.02963	0.04112
X_{124}	0.22420	−0.03906	0.08710	0.15630	−0.20330	−0.01137	0.00534	−0.09904
X_{136}	0.20591	0.19237	0.04203	0.04967	0.02966	−0.01600	0.16963	−0.05034
X_{42}	0.20308	0.02331	−0.03306	−0.07256	0.05910	0.02214	0.08560	−0.05908
X_{108}	0.20253	−0.06155	−0.00881	0.05047	−0.03234	0.16768	0.15069	−0.04344
X_{51}	0.20181	0.08366	−0.03965	0.01782	−0.07059	0.02728	−0.08054	−0.02097
X_{68}	0.20176	−0.01665	−0.01797	0.13686	0.18924	0.05817	0.06897	0.00735
X_{12}	0.19133	0.02479	0.01279	0.02607	0.08909	0.07092	−0.00086	0.05941
X_{97}	0.18598	−0.04916	−0.04393	−0.04035	−0.05636	−0.13634	0.02409	−0.01083
X_{135}	0.18505	0.01412	0.03778	0.00107	0.08253	0.02743	0.15692	−0.04902
X_{147}	0.17357	−0.02353	0.06180	−0.11565	0.04441	−0.07045	0.05416	−0.06040
X_{183}	0.01229	0.71456	0.01720	0.09053	0.04126	0.12246	−0.03982	−0.00370
X_{189}	0.04145	0.62004	0.00482	0.05886	0.03966	0.11216	0.03335	−0.07911

* 紙面관계 및 편의상 要素行列表의 一部만 수록하였음.

〈부록 3〉 선호집단과 非선호집단의 매체이용비율(「샐러드」)

(괄호 : %)

매체 및 프로그램	기사 \ 선호도 \ 집단	선호집단 매우좋아한다	선호집단 좋아한다	선호집단 그저그렇다	선호집단 싫어한다	선호집단 매우싫어한다	선호집단 合計	非선호집단 매우좋아한다	非선호집단 좋아한다	非선호집단 그저그렇다	非선호집단 싫어한다	非선호집단 매우싫어한다	非선호집단 合計
TV	뉴스 및 일기예보	96 (21.7)	228 (51.5)	105 (23.7)	5 (1.1)	3 (0.7)	443 (100.0)	35 (23.8)	69 (46.9)	34 (23.1)	7 (4.8)	2 (1.4)	147 (100.0)
	스포오츠 중계	165 (37.2)	158 (35.7)	93 (21.0)	17 (3.8)	10 (2.3)	443 (100.0)	70 (47.6)	44 (29.9)	26 (17.7)	5 (3.4)	2 (1.4)	147 (100.0)
	홈드라마	29 (6.5)	137 (30.9)	199 (44.9)	54 (12.2)	25 (5.6)	443 (100.0)	6 (4.1)	74 (50.3)	67 (45.6)	23 (15.6)	7 (4.8)	147 (100.0)
	수사 및 반공극	37 (8.4)	134 (30.2)	184 (41.5)	64 (14.4)	24 (5.4)	443 (100.0)	16 (10.9)	39 (26.5)	66 (44.9)	16 (10.9)	10 (6.8)	147 (100.0)
	사극	35 (7.9)	128 (28.9)	187 (42.2)	76 (17.2)	20 (4.5)	443 (100.0)	11 (7.5)	43 (29.3)	53 (36.1)	31 (21.1)	9 (6.1)	147 (100.0)
	퀴즈 및 게임프로	53 (12.0)	164 (37.0)	151 (34.1)	60 (13.5)	15 (3.4)	443 (100.0)	9 (12.9)	43 (29.3)	53 (36.1)	23 (15.6)	9 (6.1)	147 (100.0)
	코메디	40 (9.0)	136 (30.7)	156 (35.2)	77 (17.4)	34 (7.7)	443 (100.0)	17 (11.6)	43 (29.3)	48 (32.7)	26 (17.7)	13 (8.8)	147 (100.0)
	만화극	64 (14.4)	133 (30.0)	149 (33.6)	68 (15.3)	29 (6.5)	443 (100.0)	25 (17.0)	35 (23.8)	41 (27.9)	38 (25.9)	8 (5.4)	147 (100.0)
	외화	235 (53.0)	168 (37.9)	33 (7.4)	6 (1.4)	1 (0.2)	443 (100.0)	77 (52.4)	46 (31.3)	19 (12.9)	4 (2.7)	1 (0.7)	147 (100.0)
	쇼 및 가요프로	87 (19.6)	126 (28.4)	134 (30.2)	41 (9.3)	15 (3.4)	443 (100.0)	23 (15.6)	48 (32.7)	53 (36.1)	17 (11.6)	6 (4.1)	147 (100.0)
	주말영화	246 (55.5)	161 (36.3)	30 (6.8)	6 (1.4)	0 (0.0)	443 (100.0)	82 (55.8)	48 (32.7)	13 (8.8)	3 (2.0)	1 (0.7)	147 (100.0)
	문학작품극	211 (47.6)	176 (39.7)	45 (10.2)	8 (1.8)	3 (0.7)	443 (100.0)	67 (45.6)	48 (32.7)	21 (14.3)	9 (6.1)	2 (2.0)	147 (100.0)
	특별기획물	102 (23.0)	177 (40.0)	134 (30.2)	25 (5.6)	5 (1.1)	443 (100.0)	29 (19.7)	50 (34.0)	38 (25.9)	18 (12.2)	12 (8.2)	147 (100.0)
	애정극	43 (9.7)	114 (25.7)	193 (43.6)	72 (16.3)	21 (4.7)	443 (100.0)	15 (10.2)	25 (17.0)	56 (38.1)	34 (23.1)	17 (11.6)	147 (100.0)
라디오	뉴우스 및 일기예보	111 (25.1)	152 (34.3)	182 (41.1)	41 (9.3)	7 (1.6)	443 (100.0)	22 (15.0)	45 (30.6)	58 (39.4)	18 (12.2)	4 (2.7)	147 (100.0)
	스포오츠중계	107 (24.2)	109 (24.6)	137 (30.9)	74 (16.7)	16 (3.6)	443 (100.0)	38 (25.9)	51 (34.7)	36 (24.5)	14 (9.5)	8 (5.4)	147 (100.0)
	대중가요	73 (16.5)	173 (39.1)	141 (31.8)	43 (9.7)	13 (2.9)	443 (100.0)	31 (21.1)	45 (30.6)	51 (34.7)	15 (44.9)	5 (3.4)	147 (100.0)
	흘러간 노래	44 (9.9)	100 (22.6)	160 (36.1)	109 (24.6)	28 (6.3)	443 (100.0)	18 (12.2)	38 (25.9)	48 (32.7)	33 (22.4)	10 (6.8)	147 (100.0)

매체 및 프로그램	기사 \ 선호도 \ 집단	선호집단 매우좋아한다	선호집단 좋아한다	선호집단 그저그렇다	선호집단 싫어한다	선호집단 매우싫어한다	선호집단 合計	非선호집단 매우좋아한다	非선호집단 좋아한다	非선호집단 그저그렇다	非선호집단 싫어한다	非선호집단 매우싫어한다	非선호집단 合計
라디오	팝송	167 (37.7)	188 (42.4)	66 (14.9)	17 (3.8)	5 (1.1)	443 (100.0)	43 (29.2)	49 (33.3)	36 (24.5)	13 (8.8)	6 (4.1)	147 (100.0)
	클래식음악	92 (20.8)	182 (41.1)	120 (27.1)	37 (35.4)	12 (2.7)	443 (100.0)	19 (12.9)	55 (37.4)	49 (33.3)	17 (11.6)	7 (4.8)	147 (100.0)
	민요	21 (4.7)	76 (17.2)	176 (39.7)	131 (29.6)	39 (8.8)	443 (100.0)	8 (5.4)	23 (15.6)	55 (37.4)	48 (32.7)	13 (8.8)	147 (100.0)
	가곡	67 (15.1)	160 (36.1)	143 (32.3)	54 (12.2)	19 (4.3)	443 (100.0)	10 (6.8)	51 (34.7)	53 (36.1)	27 (18.4)	6 (4.1)	147 (100.0)
	영화음악	151 (34.1)	163 (43.6)	75 (16.9)	20 (4.5)	4 (0.9)	443 (100.0)	33 (22.4)	68 (46.3)	32 (21.8)	13 (8.8)	1 (0.7)	147 (100.0)
	심야방송	150 (33.9)	166 (37.5)	97 (21.9)	26 (5.9)	4 (0.9)	443 (100.0)	36 (24.5)	49 (33.3)	34 (23.1)	21 (14.3)	7 (4.8)	147 (100.0)
	공개방송	40 (9.0)	74 (16.7)	228 (51.5)	76 (17.2)	25 (5.6)	443 (100.0)	7 (4.8)	32 (21.8)	68 (46.3)	28 (19.0)	12 (8.2)	147 (100.0)
신문	사설	52 (11.7)	129 (29.1)	180 (40.6)	68 (15.3)	14 (3.2)	443 (100.0)	24 (16.3)	37 (25.2)	60 (40.8)	20 (13.6)	6 (4.1)	147 (100.0)
	광고	63 (14.2)	156 (35.2)	138 (31.2)	77 (17.4)	8 (1.8)	443 (100.0)	15 (10.2)	40 (27.2)	42 (28.6)	41 (27.9)	9 (6.1)	147 (100.0)
	정치	99 (22.3)	157 (35.4)	138 (31.2)	44 (9.9)	5 (1.1)	443 (100.0)	34 (23.1)	48 (32.7)	43 (29.2)	16 (10.9)	166 (4.1)	147 (100.0)
	경제	85 (19.2)	136 (30.7)	144 (32.5)	69 (15.6)	7 (1.6)	443 (100.0)	36 (24.5)	39 (26.5)	42 (28.6)	22 (15.0)	8 (5.4)	147 (100.0)
	사회	156 (35.2)	179 (40.4)	84 (19.0)	22 (5.0)	2 (0.5)	443 (100.0)	54 (36.7)	53 (36.1)	27 (18.4)	12 (8.2)	1 (0.7)	147 (100.0)
	문화	131 (29.6)	188 (42.4)	95 (21.4)	27 (6.1)	2 (0.5)	443 (100.0)	40 (27.2)	48 (32.7)	39 (26.5)	16 (10.9)	4 (2.7)	147 (100.0)
	연예, 프로	123 (27.8)	138 (31.2)	112 (25.3)	61 (13.8)	9 (2.0)	443 (100.0)	39 (26.5)	31 (21.1)	44 (29.9)	29 (19.7)	4 (2.7)	147 (100.0)
	스포오츠	201 (45.4)	111 (25.1)	78 (17.6)	43 (9.7)	7 (1.6)	443 (100.0)	89 (60.5)	29 (19.7)	20 (13.6)	6 (4.1)	3 (2.0)	147 (100.0)
	해외토픽	174 (39.3)	162 (36.6)	82 (18.5)	22 (5.0)	3 (0.7)	443 (100.0)	58 (39.5)	39 (26.5)	30 (20.4)	18 (12.2)	2 (0.7)	147 (100.0)
	연재소설	57 (12.9)	63 (14.2)	92 (20.8)	152 (34.3)	79 (17.8)	443 (100.0)	17 (11.6)	17 (11.6)	30 (20.4)	43 (29.2)	40 (27.2)	147 (100.0)
	국제외신	127 (28.7)	170 (38.4)	108 (24.4)	31 (7.0)	7 (1.6)	443 (100.0)	37 (25.2)	50 (34.0)	40 (27.2)	14 (9.5)	6 (4.1)	147 (100.0)

〈부록 4〉 선호집단과 非선호집단의 매체이용비율(「소세지」)

(괄호 : %)

매체 및 프로그램	집단 / 선호도 / 기사	선호집단						非선호집단					
		매우좋아한다	좋아한다	그저그렇다	싫어한다	매우싫어한다	合計	매우좋아한다	좋아한다	그저그렇다	싫어한다	매우싫어한다	合計
T V	뉴스 및 일기예보	107 (20.4)	278 (53.0)	124 (23.7)	12 (2.3)	3 (0.6)	524 (100.0)	24 (21.4)	50 (44.6)	32 (28.6)	4 (3.6)	2 (1.8)	112 (100.0)
	스포오츠 중계	206 (39.3)	173 (33.0)	111 (21.2)	23 (4.4)	11 (2.1)	524 (100.0)	51 (4 .5)	28 (25.0)	24 (21.4)	7 (6.3)	2 (1.8)	112 (100.0)
	홈드라마	33 (6.3)	166 (31.7)	238 (45.4)	64 (12.2)	21 (0.4)	524 (100.0)	4 (3.6)	32 (28.6)	50 (44.6)	18 (16.1)	8 (7.1)	112 (100.0)
	수사 및 반공	46 (8.8)	169 (32.3)	210 (40.1)	72 (13.7)	27 (5.1)	524 (100.0)	10 (8.9)	30 (26.8)	42 (37.5)	17 (15.2)	13 (11.6)	112 (100.0)
	사극	42 (8.0)	154 (29.4)	221 (42.2)	81 (15.5)	26 (4.9)	524 (100.0)	9 (8.0)	37 (30.1)	40 (35.7)	• (15.2)	• (8.0)	112 (100.0)
	퀴즈 및 게임프로	61 (11.6)	197 (37.6)	179 (34.2)	70 (13.4)	14 (3.3)	524 (100.0)	16 (14.2)	37 (33.0)	33 (29.5)	19 (17.0)	7 (6.3)	112 (100.0)
	코메디	45 (8.6)	150 (28.6)	198 (37.8)	92 (17.6)	37 (7.4)	524 (100.0)	11 (9.8)	36 (32.1)	39 (34.8)	24 (21.4)	12 (10.7)	112 (100.0)
	만화극	79 (15.1)	160 (30.5)	168 (32.1)	84 (16.0)	31 (6.3)	524 (100.0)	18 (16.1)	23 (20.5)	33 (29.5)	26 (23.2)	12 (10.7)	112 (100.0)
	외화	267 (52.7)	200 (38.2)	48 (9.2)	9 (17)	0 (0.0)	524 (100.0)	55 (49.0)	32 (28.6)	21 (18.8)	2 (1.8)	2 (1.8)	112 (100.0)
	쇼 및 가요프로	88 (16.8)	184 (35.1)	180 (34.4)	57 (10.9)	15 (2.8)	524 (100.0)	24 (21.5)	31 (27.7)	36 (32.1)	25 (22.3)	6 (5.4)	112 (100.0)
	주말영화	288 (55.0)	196 (37.4)	32 (6.0)	8 (1.5)	0 (0.0)	524 (100.0)	60 (53.6)	37 (33.0)	12 (10.7)	2 (1.8)	1 (0.9)	112 (100.0)
	문학작품극	247 (47.1)	214 (40.8)	53 (10.1)	7 (1.3)	3 (0.7)	524 (100.0)	46 (41.1)	50 (44.6)	8 (7.1)	5 (4.5)	3 (2.7)	112 (100.0)
	특별기획물	111 (21.2)	221 (42.2)	156 (29.8)	30 (5.7)	6 (1.1)	524 (100.0)	17 (15.2)	32 (28.6)	31 (27.7)	21 (18.8)	11 (9.8)	112 (100.0)
	애정극	46 (8.8)	133 (25.4)	226 (43.1)	93 (17.7)	26 (5.0)	524 (100.0)	9 (8.0)	20 (17.8)	46 (41.1)	19 (17.0)	18 (16.1)	112 (100.0)
라디오	뉴우스 및 일기예보	66 (12.6)	174 (33.2)	220 (42.0)	53 (10.1)	11 (2.1)	524 (100.0)	16 (14.3)	28 (25.0)	47 (42.0)	15 (13.4)	6 (5.4)	112 (100.0)
	스포오츠중계	129 (24.6)	127 (24.2)	163 (31.1)	82 (15.6)	23 (4.5)	524 (100.0)	35 (31.3)	32 (28.6)	27 (24.1)	11 (9.8)	7 (6.3)	112 (100.0)
	대중가요	79 (15.1)	209 (39.9)	169 (32.3)	55 (10.5)	12 (2.2)	524 (100.0)	21 (18.8)	39 (34.8)	31 (27.7)	15 (13.4)	6 (5.4)	112 (100.0)
	흘러간 노래	50 (9.5)	123 (23.5)	191 (36.5)	132 (25.2)	28 (5.3)	524 (100.0)	17 (15.2)	30 (26.8)	39 (34.8)	18 (16.1)	8 (7.1)	112 (100.0)

매체 및 프로그램	기사 \ 선호도 \ 집단	선호집단 매우좋아한다	선호집단 좋아한다	선호집단 그저그렇다	선호집단 싫어한다	선호집단 매우싫어한다	선호집단 合計	非선호집단 매우좋아한다	非선호집단 좋아한다	非선호집단 그저그렇다	非선호집단 싫어한다	非선호집단 매우싫어한다	非선호집단 合計
라디오	팝송	280 (53.5)	225 (42.9)	93 (17.8)	21 (4.0)	4 (0.8)	524 (100.0)	39 (34.8)	37 (33.0)	28 (25.0)	12 (10.7)	6 (5.4)	112 (100.0)
	클래식음악	96 (18.3)	229 (43.7)	143 (27.3)	44 (8.4)	12	524 (100.0)	18 (16.1)	32 (28.6)	37 (33.0)	17 (15.2)	8 (7.1)	112 (100.7)
	민요	21 (4.0)	93 (17.7)	219 (41.8)	155 (29.6)	36 (6.9)	524 (100.0)	6 (5.4)	19 (17.0)	40 (35.7)	33 (29.5)	14 (12.5)	112 (100.0)
	가곡	68 (12.8)	194 (37.0)	178 (34.0)	62 (11.8)	22 (4.4)	524 (100.0)	9 (8.0)	31 (27.7)	43 (38.4)	22 (19.6)	7 (6.3)	112 (100.0)
	영화음악	164 (31.2)	240 (45.8)	95 (18.1)	21 (4.0)	3 (0.6)	524 (100.0)	25 (22.3)	50 (44.6)	21 (18.8)	14 (12.5)	2 (1.8)	112 (100.0)
	심야방송	154 (29.4)	202 (38.4)	129 (24.5)	35 (6.7)	3 (0.6)	524 (100.0)	31 (27.7)	36 (32.1)	22 (19.6)	14 (12.5)	9 (8.0)	112 (100.0)
	공개방송	40 (7.6)	86 (16.4)	266 (50.8)	96 (18.2)	26 (5.0)	524 (100.0)	13 (11.6)	19 (17.0)	45 (40.2)	19 (17.0)	16 (14.3)	112 (100.0)
신문	사설	55 (10.5)	159 (30.3)	201 (38.4)	90 (17.2)	19 (3.6)	524 (100.0)	16 (14.3)	33 (29.5)	37 (33.0)	18 (16.1)	8 (7.1)	112 (100.0)
	광고	74 (14.2)	171 (32.6)	170 (32.4)	97 (18.5)	12 (2.3)	524 (100.0)	11 (9.8)	31 (27.7)	28 (25.0)	34 (30.4)	8 (7.1)	112 (100.0)
	정치	112 (21.4)	194 (37.0)	164 (31.3)	48 (9.2)	6 (1.1)	524 (100.0)	31 (27.7)	32 (28.6)	31 (27.7)	12 (10.7)	6 (5.4)	112 (100.0)
	경제	99 (18.9)	175 (33.4)	165 (31.5)	77 (14.7)	8 (1.5)	524 (100.0)	29 (25.9)	29 (25.9)	27 (24.1)	17 (15.2)	10 (8.9)	112 (100.0)
	사회	174 (33.2)	214 (40.8)	107 (20.4)	26 (5.0)	3 (0.6)	524 (100.0)	38 (33.9)	37 (33.0)	29 (25.9)	4 (3.6)	4 (3.6)	112 (100.0)
	문화	150 (28.6)	220 (42.0)	107 (20.4)	42 (8.0)	5 (1.0)	524 (100.0)	24 (21.4)	37 (33.0)	38 (33.9)	7 (6.3)	6 (5.4)	112 (100.0)
	연예, 프로	134 (25.6)	171 (32.6)	135 (25.8)	72 (13.7)	12 (2.3)	524 (100.0)	27 (24.1)	23 (20.5)	38 (33.9)	16 (14.3)	8 (7.1)	112 (100.0)
	스포오츠	247 (47.1)	136 (26.0)	87 (16.6)	42 (8.0)	12 (2.3)	524 (00.0)	62 (55.4)	21 (18.8)	17 (15.2)	9 (8.0)	3 (2.7)	112 (100.0)
	해외토픽	216 (41.2)	185 (35.3)	98 (18.7)	24 (4.6)	1 (0.2)	524 (100.0)	36 (32.1)	34 (30.4)	27 (24.1)	12 (10.7)	3 (2.7)	112 (100.0)
	연재소설	64 (12.2)	73 (13.9)	114 (21.8)	167 (31.9)	106 (20.2)	524 (100.0)	9 (8.0)	10 (8.9)	27 (24.1)	37 (33.0)	29 (25.9)	112 (100.0)
	국제외신	148 (28.2)	207 (39.5)	134 (25.6)	29 (5.5)	6 (1.2)	524 (100.0)	27 (24.1)	33 (29.5)	32 (28.6)	13 (11.6)	7 (6.3)	112 (100.0)