A Theory of Working Capital Investment

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I. Introduction

Working capital management involves management of the current assets: cash, marketable securities, accounts receivable, and inventories. When working capital investment is varied, the profit and risk of the firm are directly affected. Through excessive investment in current assets, the firm may have to assume undue risk if the present level of cash cannot be justified, inventory cannot be sold, accounts receivable cannot be collected, or any combination of the above. But too little investment can also be expensive and risky. If the firm does not maintain a satisfactory level of working capital, it is likely to become insolvent and may even be forced into bankruptcy.

Walker [27] has developed a descriptive working capital model which provided a theoretical framework in working capital management. But, in the literature, attempts to develop a comprehensive and operational model which explicitly covers related variables, their interrelationships, and their multi-dimensionality have seemingly failed; various independent submodels[1] have been developed to consider individual assets in isolation. With a global optimization in mind, Bierman and others [2] wrote an article in which the return from working

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1. See [1], [6], [14].

capital investment is considered as a single variable within a dynamic modeling framework, but the dynamic relationships among all components of working capital were not considered explicitly in the model; instead, the return from working capital investment was considered to be a stochastic function of the level of working capital investment.

The primary purpose of this paper is to develop criteria for an optimal investment in each current asset component within an integrated framework. For the given objective of the firm — which is to maximize the value of the firm — the risk-adjusted discount rate and streams of earnings will determine the value of the firm. In order to find the marginal contribution of working capital (WC) investment to the value of the project (or the firm), it is necessary to examine first the effects of working capital on the cost of capital of the project. Since the capital asset pricing model (CAPM) provides a schedule of required returns in the market, WC effects on the cost of capital is examined within the framework of the CAPM. Next, revenue and cost functions of WC investments are constructed and, with considerations of WC effects on the cost of capital, optimal investment criteria are derived and discussed.

II. Cost of Capital and Working Capital Investments

The capital asset pricing model is the general equilibrium models of asset prices derived by Sharpe [25], Lintner [17, 18], Mossin [23, 24], and Fama [8, 9]. Each model is an investigation of the normative Markowitz model for the equilibrium structure of asset prices and readers are referred to an excellent summary by Jensen [11] for the necessary assumptions underlying the model, its derivation, and implications in empirical studies.

The CAPM shows that, in equilibrium, the expected return, $E(\tilde{R}_i)$[2], on any asset i is determined by

$$E(\tilde{R}_i) = R_f + \beta_i [E(\tilde{R}_m) - R_f] \tag{1}$$

where R_1 = return on security i, $\frac{\tilde{P}_{t+1} - P_t + \tilde{D}_t}{P_t}$

R_t = market value of security i at time t,

D_t = dividend for period t,

R_f = return on the risk-free asset,

2. Tildes denote random variables

$\tilde{R}_m$ = return on the market portfolio, and

β_i = cov. $(\tilde{R}_i, \tilde{R}_m)$ / var $(\tilde{R}_m)$,

Equation (1) indicates that the return on security i depends entirely on the beta of the firm, because all other variables are parameters which are determined exogenously in the market.

In order to examine the impact on cost of capital of changes in working capital investment, we can observe how beta is affected as investment in working capital is varied. Given the anticipated cash flow from an investment in firm i for period t, defined as $E(X_{it}) \equiv E(P_{it+1}) - P_{it} + [E(D_{it})$: expected dividends], the expected return on security i, $E(R_i)$, can also be expressed as

$$E(R_i) = \frac{E(X_i)}{P_i}. \tag{2}$$

Then the covariance between R_i and R_m can be shown by definition as

$$\mathrm{cov}(R_i, R_m) = E\left\{\left[\frac{X}{P} - E\left(\frac{X}{P}\right)\right]\left[R_m - E\left(R_m\right)\right]\right\}$$

$$= \frac{1}{P}\,\mathrm{cov}(X, R_m) \tag{3}$$

When working capital investment is increased, the profitability of the firm will vary with other things being constant. As working capital investment is increased by W, the incremental cash flow will be $\tilde{r}$.W where the return[3] on working capital is denoted by the random variable, $\tilde{r}$. Then the firm's return with additional investment in working capital can be shown as

$$R_i^W = \frac{X_i + \tilde{r} \cdot W}{P_i}. \tag{4}$$

In equation (4), no restriction is imposed on the sign of the return, $\tilde{r}$.

If we assume that additional investment in working capital is financed entirely from debts, the covariance between the firm's return and the market return (R_m) becomes

$$\mathrm{cov}(R_i^d, R_m) = E\left\{\left[\frac{X + rW - I}{P} - E\left(\frac{X + rW - I}{P}\right)\right]\left[R_m - E(R_m)\right]\right\}$$

$$= \frac{1}{P}\left[\mathrm{cov}(X, R_m) + W\,\mathrm{cov}(r, R_m)\right]$$

$$= \mathrm{cov}(R_i, R_m) + \frac{W}{P}\,\mathrm{cov}(r, R_m), \tag{5}$$

where "I" denoted interest payment and R_i^d. the return to the firm whose additional working

3. The minimum required size of $E(\tilde{r})$ may not be equal to the required rate of return of the project.

capital is financed from debts. From equations (3) and (5), we observe that the covariance of the firm with additional investment in working capital is higher by $(W/P) \cdot \text{cov}(r, R_m)$. The difference can be rewritten by definition as

$$\frac{W}{P}\text{cov}(r, R_m) = \frac{W}{P}\rho_{rm}\sigma_r\sigma_m \tag{6}$$

where ρ_{rm} denotes correlation coefficient between r and R_m, σ_r standard deviation of r, and σ_m standard deviation of market return.

If the company product is very competitive in the market and the price is high, the product demand can be very elastic to the credit policy. When the product is highly substitutable and also similar products are readily available in the market, higher precautionary balances in inventory will perhaps be necessary in order to capture uncertain demand. Product sales, in general, will perhaps move together with economic conditions, provided that the product is not an inferior good and all other conditions are fixed. But it is very difficult to predict the level and direction of the correlation between return on incremental working capital investment and market return. Our hypothesis is that there exists no significant correlation between returns from working capital and market.

As to the working capital investment, it is possible to make any desirable changes in investment sizes at a low cost. If one anticipates, however, a lower return on working capital investment under a specific condition, he may have to reduce the investment size, with other things being constant, in order to maintain at least a desirable marginal return, and vice versa. Consequently the variability of return on working capital and ρ_{rm} will likely be insignificant. When the return, $\tilde{r}$, is very uncertain under all circumstances, the correlation coefficient will most likely be close to zero. In addition, the multiplier (W/P) in equation (6) can also be very small. As a result, the second term in equation (5) will be very small.

When working capital is financed fully from equities, the analysis is not much different from the above. Then the covariance will be

$$\begin{aligned}\text{cov}(R_i^e, R_m) &= E\left\{\left[\frac{X+rW}{P+W} - E\left(\frac{X+rW}{P+W}\right)\right]\left[R_m - E(R_m)\right]\right\} \\ &= \frac{1}{P+W}\text{cov}(X, R_m) + \frac{W}{P+W}\text{cov}(r, R_m)\end{aligned}$$

$$= \frac{P}{P+W}\left[\frac{W}{P}\operatorname{cov}(X, R_m) + \frac{W}{P}\operatorname{cov}(r, R_m)\right]$$
$$= \frac{P}{P+W}\left[\operatorname{cov}(R_t, R_m) + \frac{W}{P}\operatorname{cov}(r, R_m)\right] \qquad (7)$$

Equation (7) is different from equation (5) only by scale. Since the multiplier (W/(P + W)) is always smaller than 1, the covariance term will always be smaller than that from debt financing.

Working capital policy also affects the overall liquidity of the firm, but the liquidity of individual firms itself would not have any impact on shareholders wealth. Van Horne [20] argued effectively that, in the perfect capital markets, the degree of liquidity of a firm would be a matter of indifference to shareholders because shareholders themselves can also manage their portfolios in such a way so as to satisfy their utility for liquidity.

In conclusion, we may say that impacts on cost of capital of changes in working capital investment will most likely be negligible, *ceteris paribus*. In the following section, optimality criteria for working capital investment are examined with cost of capital given, and their implications are discussed.

III. Optimality Conditions

Working capital is different in nature from fixed assets. Fixed assets are lumpy in the sense that they involve the commitment of a larger amount of funds to an asset over a longer time period. But current assets can be increased or decreased in smaller units as may be desired. Consequently, working capital investments in this paper are handled separately with the fixed asset investment given, but the investment decision is made based upon the level of contribution to the overall value of the firm on which all other investment decisions are evaluated.

As the investment level of individual working capital components is varied with the fixed asset investment given, the revenue of the firm is directly affected, *ceteris paribus*. Frequently, sales volumes are significcantly affected by relaxing credit policy, which in return will increase the investment level in accounts receivable. As inventory level is increased, stock-outs in general will be decreased, and consequently sales volumes will be increased. The amount of cash on hand may not have a direct impact on sales, but the cost of material and labor can be reduced; material and labor can often be purchased at a lower cost in cash than otherwise.

The revenue from working capital investments also depends upon economic conditions, availability of the product, and other exogenous factors that are not controllable by the company. Consequently, the revenue of the firm i from working capital investment in period t, which is denoted by Y_{it}, can be expressed as random variables whose probability distributions depend on the cash (c_t), marketable securities (m_t), accounts receivable (a_t), and inventories (i_t). In order to give a simple formulation of these relationships, it is assumed that the uncertainty can be parameterized by the random variable 0 which indicates the state of the world. The stochastic revenue function is

$$Y_{it}(\theta) = f_{it}(c_t, m_t, a_t, i_t, \theta \mid F) \tag{8}$$

where F denotes the level of fixed-asset investment. $Y_{it}(\theta)$ are revenues of the firm i in the given state θ, which are generated from investments in working capital. The functions $Y_{it}(\theta)$ are assumed to be strictly nondecreasing, concave functions of c_t, m_t, a_t, and i_t.

The cost to the firm also changes as the level of working capital investment varies. The cost X_{it} are random variables whose probability distributions depend upon the investment level in each component of working capital. This is mainly because the cost of working capital depends upon exogenous variables such as inflation and other economic conditions. The stochastic cost function is

$$X_{it}(\theta) = g_{it}(c_t, m_t, a_t, i_t, \theta \quad F) \tag{9}$$

where θ denotes the state of the world which is assumed to be discrete. The cost functions g_{it} are strictly increasing, convex functions of investment level in each component of working capital.

The profit function $P_{it}(\theta)$ can be expressed as the difference between the above two equations:

$$P_{it}(\theta) = Y_{it}(\theta) - X_{it}(\theta) \tag{10}$$

The equation (10) represents a family of profit functions — one for each "state of the world".

The expected profit can now be expressed as

$$P_{it} = \sum_{\theta} P_{it} (c_t, m_t, a_t, i_t, \theta \mid F) f_i(\theta) \tag{11}$$

where $f_i(\theta)$ is the probability density function which is assumed to be identical for each period. Since we can suppress the functional dependence of random variables on θ by using the expectation operator, the equation (11) can be expressed compactly as

$$P_{it} = E[P_{it} (c_t, m_t, a_t, i_t \mid F)] \tag{12}$$

Then the present value of wealth will be

$$P_i = \sum_{t} P_{it} (1+k)^{-t} \tag{13}$$

where k is the cost of capital to the firm i. In the previous section we found that working capital investment has little impact on the cost of capital within the framework of the CAPM. Even under the imperfect market, cost of capital may not be particularly sensitive to the working capital policy as long as bankruptcy risk is not significantly high in the near future. Consequently, it is assumed that the cost of capital is exogenously determined in this model.

The budget available for working capital investment is limited, like other capital budgets, at the given cost of capital which the company can tolerate. The size of the budget for each period is dependent upon the line of credit, availability of equity capital, anticipated amount and timing of cash flows from the fixed asset investment, and duration of working capital investment required. For the given budget limits b_t ($t = 1 \ldots T$) which are determined exogenously, the budget constraints can be expressed as

$$c_t + m_t + a_t + i_t \leq b_t, \quad \text{for} \quad t = 1, \ldots, T \tag{14}$$

and the non-negativity constraints are as follows:

$$c_t, m_t, a_t, i_t \geq 0, t = 1, \ldots, T \tag{15}$$

If it is necessary for a company to maintain a certain level of investment in each component of working capital, the non-negativity constraint can be modified to reflect the lower limit of investment.

The size of investment for company i is determined by the attempt to maximize $P_i = \sum_t P_{it}$ $(c_t, m_t, a_t, i_t)(1+k)^{-t}$, subject to the budget constraints of company i and non-negativity constraint for each period. Now we can form the Lagrangean

$$L = P_i + \sum_t \lambda_t (b_t - c_t - m_t - a_t - i_t) \qquad (16)$$

where λ_t (for t = 1... T) are Langrangean multipliers. Then the following Kuhn-Tucker conditions will hold at $(c^*_t, m^*_t, a^*_t, i^*_t, \lambda^*_t;\ t = 1 \ldots T)$[4].

$$\frac{\partial L}{\partial c_t} = \frac{\partial P_i}{\partial c_t} - \lambda_t^* \leq 0, \qquad \text{for all } t$$

$$\frac{\partial L}{\partial c_t} c_t^* = \left(\frac{\partial P_i}{\partial c_t} - \lambda_t^*\right) c_t^* = 0, \qquad \text{for all } t$$

$$c_t^* = \geq 0, \qquad \text{for all } t$$

$$\vdots$$

$$\frac{\partial L}{\partial \lambda_t} = b_t - c_t^* - m_t^* - a_t^* - i_t^* \geq 0, \qquad \text{for all } t$$

$$\lambda^* \frac{\partial L}{\partial \lambda_t} = \lambda_t^* (b_t - c_t^* - m_t^* - a_t^* - i_t^*) = 0, \qquad \text{for all } t$$

$$\lambda_t^* \geq 0, \qquad \text{for all } t.$$

(17)

A certain level of minimum cash balance is frequently necessary for a company to maintain routine operations. It may be impossible for a company to eliminate[5] inventories entirely until it ceases to operate. A certain level of receivables balance may also be necessary just for routine operations. Due to uncertainty in cash demand, it may be strongly desirable to maintain a positive balance in the marketable securities. Under these circumstances, the cost of not maintaining the minimum level will be very high. Consequently, the balance at the optimum solution will be at least equal to or greater than the minimum required level whether these minimum levels are imposed as constraints on the problem or not.

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4. "*" denotes optimum solution.

5. If the products are sold exclusively via mail-orders, this company may not need to maintain any inventory at all.

If $c^*_t > 0$, from equation (17) we have

$$\frac{\partial P_i}{\partial c_t} = \lambda^*_t, \text{for all } t. \tag{18}$$

By using the same analogy, we finally obtain

$$\frac{\partial P_i}{\partial c_t} = \frac{\partial P_i}{\partial m_t} = \frac{\partial P_i}{\partial a_t} = \frac{\partial P_i}{\partial i_t} = \lambda^*_t, \text{for all } t. \tag{19}$$

The above equation simply states that at the optimum level, the marginal return of individual working capital components must be equal to the shadow price λ^*_t for each period.

The shadow price λ_t is defined as

$$\lambda_t = \frac{\partial P_i}{\partial b_t} \tag{20}$$

which can be interpreted as the change in the optimal value of the objective function as the right-hand side constants in the constraints change. In other words, λ_t is the marginal return of the incremental investment in working capital. From equation (17) we can see that λ^*_t must be zero when $b_t > c^*_t + m^*_t + a^*_t + i^*_t$. This indicates that marginal returns of additional investment are zero at the optimal solution if the solutions are achieved within budget constraints for each period. At these levels, any change in working capital balance will decrease the objective function value.

If $b_t = c^*_t + m^*_t + a^*_t + i^*_t$, then λ^*_t, then λ^*_t is either greater than or equal to zero. When $\lambda^*_t = 0$, optimal solutions are still obtained within the budget limits. But, when λ^*_t are positive, the budget constraint for that period is binding. Then it is most likely that λ^*_t is not identical for each period, since identical λ_t ($t = 1 \ldots T$) imply the same marginal return on incremental investment in working capital for all periods. This indicates that a better solution can be achieved by reallocating budgets intertemporally. Without increasing total budgets available for the predetermined management horizon, total budgets can be reallocated until all λ^*_t are equal; consequently, the objective function value will be improved. The optimality condition in equation (19) can now be rewritten as

$$\frac{\partial P_i}{\partial c_t} = \frac{\partial P_i}{\partial m_t} = \frac{\partial P_i}{\partial a_t} = \frac{\partial P_i}{\partial i_t} = \lambda^*, \text{for all } t \tag{21}$$

which indicates that all marginal returns of individual components of working capital investment are equal.

When λ^* is still positive, managers must consider raising additional funds because the marginal return of additional investment is positive. With considerations of all costs for raising additional funds, if the model provides positive λ^*, the company budget must be expanded up to the point where λ^* equals zero, *ceteris paribus.*

IV. Concluding Remarks

This paper investigated theoretically the optimality criteria for working capital investments by constructing stochastic revenue and cost functions. In this model, individual components of working capital are explicitly considered with an integrated framework, and dynamic interrelationships over the given investment horizon are also implied in the stochastic revenue and cost functions. When costs and revenues associated with individual working capital balances are non-linear, they can also be handled within the confines of the above model by using piece-wise linear programming techniques.

Optimal criteria which have been defined by λ_t are also precise and operational, and can therefore be applicable to the working capital investment decision whether the independent variables are correlated or not. In order to build an operationally feasible model, it is strongly desired to have a multi-period dynamic model. Due to the fact that the model must be run repeatedly, the importance of the computational efficiency cannot be overemphasized; otherwise, the model will be of little value.

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