Optimizing Supply Chain Structure To Embrace Omnichannel Marketing*
옴니채널 추진에 따른 최적 공급사슬 구조

Hag-Soo Kim(First Author)
Dept. of Bus. Admin. Gyeonsang National University
(*bbhskim@naver.com*)

We investigate the optimal supply chain structure in inventory location with the advance of omnichannel in the retail industry. Based on a dyadic supply chain consisting of a supplier and buyer(s) considering asymmetric information and flexible production lead times, we show that a dichotomous inventory location, combining VMI (Vendor Managed Inventory) and RMI(Retailer Managed Inventory), is optimal for the whole system in the presence of nonstationary demand across periods and asymmetric information between the suppplier and the buyer(s). Next, on the request of mass merchandisers to increase material availability under omnichannel initiatives, we show that more investment in reducing the production lead time is necessary when customer demands are less correlated across periods. It seems that such guideline for the investment in material availability can be more valuable with an increasing trend of omnichannel initiatives to revamp consumer experience in an uncertain market environment.

Key Words: Omnichannel, Supply Chain, Vendor Managed Inventory, Asymmetric Information

Ⅰ. Introduction

With digital technologies, the retailing industry evolves toward a seamless omnichannel retailing experience, and the distinctions between physical and online vanish, as argued in Brynjolfsson et al. (2013). Wal-Mart's omnichannel shows that the company is making a range of initiatives, from the pickup, delivery, and data analytics to its premium online subscription service and its curated e-commerce channel and making additional initiatives, and such an omnichannel requires another range of initiatives to provide increased end-to-end visibility in material availability despite the organizational independence of the suppliers. It seems that such a hybrid model is changing

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the landscape in the retail industry radically while leading manufacturers to reconsider their supply chain distribution strategies due to the increase in operating costs for the omnichannel.

Implementing the omnichannel is a highly complicated task as it needs integration of service for shoppers in their search and buying process and a broader perspective on the issue of optimizing the supply chain structure and inventory management. In Walmart (2019) and WalmartMediaGroup (2020), we can observe Wal-Mart requests its suppliers to provide increased end-to-end visibility in material availability for an integrated service. Gallino et al. (2016) claimed that the performance of the whole distribution system under omnichannel initiative could differ significantly due to the inventory fulfillment for customers' streamlined experience of customers, and it indicates that the omnichannel dynamics of Wal-Mart can be more complicated than that under Vendor Managed Inventory (VMI) which is well-known as one of the pioneering initiatives of Wal-Mart in the past. Despite the evidence in Herhausen et al. (2015) showing increasing outcomes via perceived service quality due to the online and offline integration through the omnichannel, we expect that omnichannel dynamics can cause significant conflict among all entities in the channel as the necessity of improved material availability to embrace the omnichannel marketing requests suppliers to have more responsibility in managing inventory, especially in highly uncertain market conditions.

From River Logic (2022), we can find the trend of distribution channels adopting multiple regional distribution centers, relying less on large centralized ones, and selecting third-party distributors to supply stores and ship directly to customers. Under such a supply chain, the previous pilot projects of VMI have received a great deal of attention among the academia and the industries in the past as the means to alleviate the harmful double marginalization effects under Retailer Managed Inventory (RMI). However, on the issue of who benefits from VMI, we still have conflicting evidence. For example, Bourland et al. (1996), Cachon and Fisher (2000), Gavirneni et al. (1999), Lee et al. (2000), and Raghunathan and Yeh (2001) claimed that the manufacturer is the primary beneficiary, whereas Baljko (2003) reported that about 60% of electronics suppliers experience a cost surge after VMI implementation while about 70% of their customers experience a cost reduction. Furthermore, Schenck and McInerney (1998) claimed that drivers for the success and failure of VMI are controversial among presses despite the remarkable success stories of VMI. Considering that the omnichannel dynamics are more complicated than that under VMI, as shown previously, mass merchants are still trying to figure out what is the right strategy to over-come the challenges they face in embracing omnichannel marketing.

We propose a single supplier-multiple buyer model that allows a hybrid supply chain (VMI or RMI or an intermediate mixture of them) to investigate the optimal supply chain structure to embrace omnichannel marketing driven by mass merchants. As the basic assumptions we allow that customer demands, occurring only at the buyers' sites, are stochastic and follow non-stationary auto-correlated AR(1) processes. For simplicity of analysis, we assume independent demands across buyers. The supplier can meet buyers' orders within a fixed production lead time. All the supply chain participants are following the base-stock-policy. The supplier and all the buyers have inventory holding and shortage penalty costs per unit per period of time, based on their net inventory levels at the end of a period, and there exists no fixed cost for the shipment from the supplier to each buyer. The supplier makes a contract with each buyer about the distribution lead time and the resulting inventory locations in the supply chain and the supplier can invest in her production process to reduce the production lead time at the supplier's factory. As the key feature of our model, we consider the buyers' decision to share his own private information, whereas we do not consider abusing of information, recipients of information sharing benefits, and the confidentiality of the shared information which are regarded as hurdles in information sharing, as noted in Lee and Whang (1999).

Our approach is similar to studies on the cooperation between the vendor and buyers for improving the performance of inventory control such as Goyal (1988) and Lu (1995) for joint economic lot sizing decisions for single vendor single buyer model, and Jalbar et al. (2005) for single vendor multiple buyers models. The critical difference lies in the stochastic non-stationary demand and flexible production lead time, which seems to be more realistic in representing the VMI in practice and highly important affecting the performance of the supply chain in a broad sense. We can find a similar structure in Cachon and Fisher (2000), addressing the issue of information sharing between multiple identical retailers and a single uncapacitated supplier, and Graves (1996), on a multi-echelon inventory model based on Poisson distributions. However, our model is more general than Cachon and Fisher (2000) as we allow non-identical retailers in a dyadic supply chain, and it is less restrictive compared to the Poisson distribution of Graves (1996). In addition, as our model allows the analysis of the effect of flexible production lead time, it reflects the requests of retailers to provide increased end-to-end visiibility in material availability for an integrated service for consumers when mass merchants are engaged in omnichannel initiatives.

The single supplier-multiple buyer structure of our paper is similar to Barnes-Schuster et al. (2006) assuming random demand in each period with independent and identical distribution under based stock policy. The key difference is the asymmetry of the information under a nonstationary demand which corresponds to a critical feature in VMI by measuring the information sharing effect under autocorrelated AR(1) processes, as in Lee et al. (2000). Furthermore, we allow the flexibility of production lead time in a more general business environment which seems to be one of the key feature in the implementation of omnichannel initiatives as we can observe evidence in Walmart (2019) and Walmart MediaGroup (2020). In addition, as we do not take into account specific restrictions in the supply chain, like (z, Z) condition in Fry et al. (2001). However, the assumptions are more general than previous studies such as Chen et al. (2001) focusing on VMI systems with independent and competing retailers. More related studies on different issues in VMI such as the ratio of the order costs and the carrying charges of the supplier with different demand assumptions can be found in Zhang et al. (2007) and Yao et al. (2007).

Ⅱ. The Model

We consider a dyadic supply chain of a single supplier and a buyer as the foundation of the analysis. Customer demands occur only at the buyers. The demand process $\{D_t\}$ at each buyer is a nonstationary autocorrelated process such that

$$D_t = d + \rho D_{t-1} + \epsilon_t, \tag{1}$$

where $d > 0$, $-1 < \rho < 1$, and all the ϵ_ts are independent and identically distributed normally distributed with mean zero and variance σ^2. To have negligible probability of a negative demand, σ is assumed to be significantly smaller than d. The demand process of each buyer is independent of each other's. Let L and l denote the supplier's production lead time and the delivery time from the supplier to a buyer, respectively, and functions Φ and ϕ are the standard normal distribution and density functions, respectively. Assuming that both the supplier and the buyers review their inventories periodically and replenish them with goods from the upstream sites every period using a base-stock policy, as in Raghunathan (2003), we set the sequence of events as follows. First, each buyer observes demand D_t realized at period t, reviews its inventory level, and then places an order of size Y_t to the supplier. Second, the supplier delivers the goods ordered at period $t-l$ to each buyer. Third, the buyer receives the goods ordered at period $t-l$ and fulfills demand D_t at period t. Fourth, the supplier completes production of goods ordered at period $t-L$ and initiates production on orders placed by the buyer at period t. Finally, on-hand and on-order inventories are counted, and costs are calculated.

We assume that the supplier's production lead time is known and independent of order size. To simplify the analysis, we also assume that the supplier can meet all orders from the buyers by expediting its production process. No fixed cost is incurred when placing an order and unit inventory holding and delivery expedition costs are constant over time as Lee et al. (2000). The terms h and p denote the unit inventory holding cost and the delivery expedition cost per period for the buyer, respectively. H and P denote the unit inventory holding cost for the supplier and the expedition cost per period for the supplier from an alternative source, respectively. We assume that both the supplier and the buyer (or buyers) can expedite the process by paying the additional cost to fulfill all customer demands without a fixed cost for simplicity of analysis. The supplier delivers the entire amount of goods ordered by the buyer after a fixed delivery lead time. By negotiating to increase the delivery lead time to the buyer, the supplier can reduce her demand risk.

2.1 The Buyer Model

The buyer has an inventory problem with a fixed delivery lead time l and a demand process $\{D_t\}$. From recursive equation (1), the total demand during the delivery lead time is written as

$$\sum_{i=1}^{l} D_{t+i} = \frac{1}{(1-\rho)}\{d \sum_{i=1}^{l}(1-\rho^i) + \rho(1-\rho^l)D_t\}$$
$$+\epsilon_{t+l} + (1+\rho)\epsilon_{t+l-1} + \cdots + (1+\rho+\rho^2+\cdots+\rho^{l-1})\epsilon_{t+1}.$$

Therefore, for a given demand D_t at period t, the total demand until the delivery, that is, during periods $t+1, t+2, \ldots, t+l$, has a normal distribution $F_t(x \mid D_t)$ with mean m_t and variance v_t such that

$$m_t = E(\textstyle\sum_{i=1}^{l} D_{t+i} \mid D_t) = \sum_{j=1}^{l}(1-\rho^j)\frac{d}{1-\rho} + D_t \ \frac{\rho(1-\rho^l)}{1-\rho} \ \ and$$

$$v_t = Var(\textstyle\sum_{i=1}^{l} D_{t+i} \mid D_t) = \sigma^2 \sum_{j=1}^{l}(1-\rho^j)^2/(1-\rho)^2 \ \equiv \sigma^2 v.$$

Let S_t is the buyer's optimal order-up-to level. Then, the sum of the expected holding and expedition costs at period $t+l$ is minimized for $S_t = m_t + k\sigma\sqrt{v}$, where $k \equiv \Phi^{-1}(p/(p+h))$. We then have the buyer's optimal order quantity at period t as

$$Y_t = D_t + (S_t - S_{t-1}) = D_t + \frac{\rho(1-\rho^l)}{(1-\rho)}(D_t - D_{t-1}). \quad (2)$$

The sum of the buyer's expected inventory holding and expedition costs, C_t^b, can be obtained as

$$C_t^b \equiv p\int_{S_t}^{\infty}(x-S_t)\,dF_t(x|D_t) + h\int_{-\infty}^{S_t}(\mathrm{S_t}-x)\,dF_t(x|D_t)$$
$$= \sigma\sqrt{v}\ [(h+p)L(k)+hk] \qquad (3)$$
$$= \sigma\sqrt{v}\ \phi(k)(h+p), \qquad (4)$$

where $L(x) \equiv \int_x^{\infty}(z-x)\,d\Phi(z)$. The result in equation (3) holds for any value of k when S_t is set as $S_t = m_t + k\sigma\sqrt{v}$ for given values of p and h, and the result in equation (4) holds specifically for given $k = \Phi^{-1}(p/(p+h))$. We will use them for the analysis of the asymmetric information case and the full information case, respectively. It is easy to show that C_t^b is monotone increasing and concave in $l \in [0, \infty)$.

2.2 The Supplier Model

We assume that the supplier knows the distribution of the demand process $\{D_t\}$ for each buyer. The supplier's production decision process is given as follows. After the supplier receives an order Y_t from a buyer, it immediately initiates production at period t to bring its inventory position up to level T_t. The order will be ready for shipment at the beginning of period $t+L+1$. As the supplier can avoid the risk from inventory during the delivery lead time l set by the buyer, the supplier does not need to keep inventory to cover the orders during its whole production lead time L, as shown in Lee et al. (2000). Therefore, the order-up-to level T_t covers only the orders during the time $\tau \equiv L - l$, which can be named its effective production lead time, similarly to Barnes-Schuster et al. (2006). However, different from the stationary model, the supplier anticipates the total autocorrelated demand during its effective production lead time.

The supplier's total shipment quantity over its effective production lead time is $B_t = \sum_{i=1}^{\tau+1} Y_{t+i}$. To determine the conditional mean and variance of B_t given a buyer's order Y_t, we write B_t in terms of Y_t. By repetitively applying equations (1) and (2), we have $Y_{t+i} = d(1-\rho^i)/(1-\rho) + \rho^i Y_t + \epsilon_{t+i}(1-\rho^{l+1})/(1-\rho) + \sum_{k=1}^{i-1}\rho^{l+k}\epsilon_{t+i-k} - \epsilon_t(\rho^i(1-\rho^l))/(1-\rho)$, $i = 1, 2, \ldots$, and we have

$$B_t = \frac{d}{1-\rho}\left\{(\tau+1) - \frac{\rho(1-\rho^{\tau+1})}{1-\rho}\right\} + \left(\frac{\rho(1-\rho^{\tau+1})}{1-\rho}\right)Y_t$$
$$+ \frac{(1-\rho^{l+1})}{1-\rho}\epsilon_{t+\tau+1} + \frac{1}{1-\rho}\sum_{i=1}^{\tau}(1-\rho^{\tau+l+2-i})\epsilon_{t+i}$$
$$- \frac{\rho(1-\rho^{\tau+1})(1-\rho^l)}{(1-\rho)^2}\,\epsilon_t. \qquad (5)$$

In order to determine the supplier's order-up-to level T_t that minimizes the sum of expected inventory holding and expedition costs at period $t+\tau+1$, the supplier needs to identify the probability distribution of B_t. We assume that the supplier knows both the buyer's order quantity Y_t and the error term ϵ_t when it determines the order-up-to level

T_t. From (5), we can infer that the total shipment quantity over the supplier's lead time, $(B_t | \epsilon_t)$, has a normal distribution F_t' with mean M_t' and variance $\sigma^2 V'$ such that

$$M'_t = \{(\tau+1) - \frac{d}{(1-\rho)}\frac{\rho(1-\rho^{\tau+1})}{1-\rho}\} + \frac{\rho(1-\rho^{\tau+1})}{1-\rho}Y_t$$
$$- \frac{\rho(1-\rho^{\tau+1})(1-\rho^{l})}{(1-\rho)^2}\epsilon_t \text{ and}$$
$$V' \equiv \sum_{i=l+1}^{L+1} (1-\rho^i)^2 / (1-\rho)^2.$$

We note that the two terms M_t' and V' are functions of l due to $r = L - l$. So, the supplier's optimal order-up-to level is $T_t'^* = M_t' + K\sigma\sqrt{V'}$.

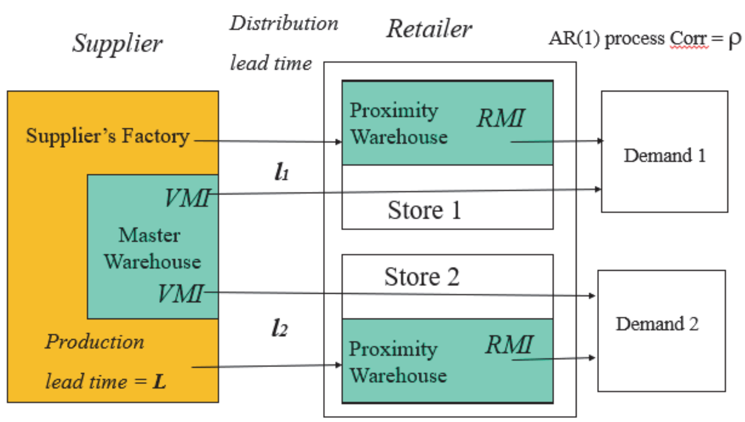
For given $K = \Phi^{-1}(P/(P+H))$, the sum of the supplier's expected inventory holding and expedition cost at period $t+\tau+1$ under full information sharing, $C_t'^s$, is

$$C_t'^S \equiv E_{\epsilon_t} P \int_{T_t'^*}^{\infty} (x - T_t'^*) dF_t'(x)$$
$$+ H \int_{-\infty}^{T_t'^*} (T_t'^* - x)\, dF'_t(x)\}$$
$$= \sigma\sqrt{V'}[(H+P)L(K) + HK] \quad (6)$$
$$= \sigma\sqrt{V'}\,\phi(K)(H+P). \quad (7)$$

Similarly as in the buyer model, the result in equation (6) holds for any value of K when $T_t'^* = M_t' + K\sigma\sqrt{V'}$ for given values of P and H, and the result in equation (7) holds specifically for given $K = \Phi^{-1}(P/(P+H))$. We will use them for the analysis of the asymmetric information case and the full information case, respectively. $C_t'^s$ is independent of M_t' and D_t, and is monotone decreasing and concave in $l \in [0, L+1]$.

III. The Optimal Supply Chain Structure

Based on the mathematical model in the previous section, we consider a dyadic supply chain of a single supplier and n buyers under the omnichannel environment. In such an environment (see Figure 1), each of the buyers represents stores in different locations belonging to a giant retailer, like Wal-Mart shown in River Logic (2022), adopting the omnichannel strategy allowing customers to visit and pick up items or the store come to them when products are delivered, as reported in Bell et al. (2014). All buyers are assumed to have identical cost parameters as Lee et al. (2000) and Raghunathan (2003), as omnichannel retailers charge the same price for the items sold online and offline, regardless of stores. However, each buyer in a different location faces a demand with different demand variability as $\sigma_1 \le \sigma_2 \le \cdots \le \sigma_n$. We let subscript a denote each buyer. From now on, we investigate the structural properties of the supply chain under two different information sharing scenarios.



〈Figure 1〉 Omnichannel Retailing with Two Stores in Different Locations

3.1 Full Information

We first consider the case where the supplier has full information on the demand at each buyer and each buyer a negotiates with the supplier independently for the delivery lead time $l_a \in [0, L+1]$. The supplier uses a base-stock-policy and its demand at each period t has a normal distribution with standard deviation $\Psi' \equiv \sqrt{\sum_{a=1}^{n} V'^{a}\sigma_a^2} = \sqrt{\sum_{a=1}^{n}\left(\frac{\sigma_a}{1-\rho}\right)^2 \sum_{i=l_a+1}^{L+1}(1-\rho^i)^2}$ and mean $\Upsilon_t' \equiv \sum_{a=1}^{n} M'_{ta}$, where M'_{ta} and $V_a'\sigma_a^2$ are the mean and variance of the total shipment quantity to buyer a, respectively, over the supplier's effective lead time as shown in the previous section. Then, the supplier's optimal order-up-to level is $T_t'^{*} = \Upsilon_t' + K\Psi'$ and, using the results in equations (4) and (7), the total supply chain cost $TC_t'(l_1, l_2, \dots, l_n)$ can be computed as

$$TC'_t(l_1, l_2, \dots, l_n) = \sum_{a=1}^{n}\phi(k)(h+p)\frac{\sigma_a}{1-\rho}\sqrt{\sum_{j=1}^{l_a}(1-\rho^j)^2} + \phi(K)(H+P)\Psi'.$$

Now, we characterize the total supply chain cost function, and examine the optimal structure of the supply chain, that is, where the inventories should be positioned. Let $\beta \equiv \phi(k)(h+p)/\phi(K)(H+P)$, $\beta_{k,k'} \equiv \left(\sqrt{\sum_{a=1}^{k}\sigma_a^2} - \sqrt{\sum_{a=1}^{k'}\sigma_a^2}\right) / \sum_{a=k'+1}^{k}\sigma_a \ \forall k > k'$ and $\sigma_1 \le \sigma_2 \le \cdots \le \sigma_n$, then we have the following results.

Proposition 1 1) $TC_t'(l_1, l_2, \dots, l_n)$ is concave in $l_a \in [0, L+1] \ \forall a = 1, \dots, n$, and the minimum exists at l_a^* corresponding to one of the extrema of $l_a \in [0, L+1] \ \forall a = 1, \dots, n$.

2) The total supply chain cost is minimized under the following three conditions given as: (i) The supplier hold inventory for buyers 1 through m, and the buyers $m+1$ through

n hold their own inventories (i.e. $l_a = 0$ $\forall a = 1, \ldots, m$ and $l_a = L+1$ $\forall a = m+1, \ldots, n$) if $max_{k=0,\ldots,m-1}\{\beta_{m,k}\} \leq \beta \leq min_{k=m+1,\ldots,n}\{\beta_{k,m}\}$ for some $m \in [1,\ldots,n-1]$; (ii) only the buyers hold their own inventories (i.e. $l_a = L+1$ $\forall a = 1, \ldots, n$) if $\beta \leq min_{k=1,\ldots,n}\{\beta_{m,k}\}$ for $m = 0$; and (iii) only the supplier hold inventory (i.e. $l_a = 0$ $\forall a = 1,\ldots,n$) if $\beta \geq max_{k=0,\ldots,n-1}\{\beta_{m,k}\}$ for $m = n$.

The extreme point $l_a = 0$ corresponds to the case when the supplier is keeping inventory for buyer a as in VMI program. The other extreme point $l_a = L+1$ is the case when buyer a keeps its own inventory as in RMI program. Therefore, Proposition 1 confirms that the results of Barnes-Schuster et al. (2006) for a two-retailer model can be generalized to more than three-retailer cases in a more general environment allowing information asymmetry. Our key findings, as compared with those of Barnes-Schuster et al. (2006), are paraphrased as follows.

- When $n = 1$, we have $min_{k=1,\ldots,n}\{\beta_{k,0}\} = max_{k=0,\ldots,n-1}\{\beta_{n,k}\} = 1$. Therefore, the buyers should hold their inventory (RMI is preferred) if $(\phi(k)(h+p))/(\phi(K)(H+P)) \geq 1$ and vice versa.
- When $n = 2$, we have $max_{k=0,\ldots,n-1}\{\beta_{n,k}\} = min_{k=1,\ldots,n}\{\beta_{k,0}\} = \sqrt{(\sigma_1^2+\sigma_2^2)/(\sigma_1+\sigma_2)}$. Therefore, the buyers should hold their inventory (RMI is preferred) if $(\phi(k)(h+p))/(\phi(K)(H+P)) \geq \sqrt{(\sigma_1^2+\sigma_2^2)/(\sigma_1+\sigma_2)}$ and *vice versa*. It confirms that the result in Barnes-Schuster et al. (2006) for a two-buyer case holds under autocorrelated nonstationary demand as well. In the presence of a power retailer and a fringe in a competitive market, showing $p \gg P \gg \max\{h, H\}$ in general due to the high cost when inventory is not available at the retailers' sites than that of the supplier, RMI is recommendable. However, RMI may still be recommendable even if the retailer's expedition cost p is less than that of the supplier represented as P.
- Proposition 4 of Barnes-Schuster et al. (2006) for $\beta < 1$ corresponds to our result for the case when $\beta_{m,m-1} \leq \beta \leq \beta_{m+1,m}$ for some m. However, the supplier may need to hold inventory even for $\beta < 1$ if $\beta \geq \max_{k=0,\ldots,n-1}\{\beta_{n,k}\}$.
- Either all buyers or only the supplier should hold inventory (a mixture of RMI and VMI is not allowed) if $\sqrt{\Sigma_1^k \sigma_i^2 / \Sigma_1^n \sigma_i^2} \geq \Sigma_1^k \sigma_i / \Sigma_1^n \sigma_i$ $\forall k = 1, \ldots, n$. For a special case when $\sigma_i = \sigma$ $\forall i$, the supplier should hold all inventory if $\beta \geq \sqrt{\Sigma_1^n \sigma_i^2} / \Sigma_1^n \sigma_i$. Otherwise, the buyers should hold all inventory. As $\sqrt{\Sigma_1^n \sigma_i^2} / \Sigma_1^n \sigma_i < 1$, the supplier may need to hold all inventories for the buyers even if the supplier has a higher sum of the inventory holding and expedition costs than that of the buyers.

3.2 Asymmetric information

We now suppose that the supplier receives information only on the buyer's order quantity Y_{at} and the error term ϵ_{at} is unknown to the supplier when it determines its order-up-to level. We let $\bar{\epsilon}_t \equiv (\epsilon_{it}, \ldots, \epsilon_{nt})$. The effective production lead time for each buyer a is $\tau_a = L - l_a$. From equation (5), the supplier knows that the total shipment amount for each buyer a during its effective lead time, B_t^a, has a normal distribution with mean M_{ta} and variance $\sigma_a^2 V_a$ such that $M_{ta} = M'_{ta} + \epsilon_t \rho(1-\rho^{\tau_a+1})(1-\rho^{l_a})/(1-\rho)^2$ and $V_a = V'_a + \rho^2(1-\rho^{\tau_a+1})^2(1-\rho^{l_a})^2/(1-\rho)^4$. Therefore, the supplier has total demand from the buyers that has a normal distribution F_t with mean Υ_t and standard deviation Ψ such that $\Upsilon_t \equiv \sum_{a=1}^{n} M_{ta}$ and $\Psi = \sqrt{\sum_{a=1}^{n} V_a \sigma_a^2}$. The supplier's optimal order-up-to level that minimizes the expected inventory holding and expedition costs for demand distribution F_t is $T_t^* = \Upsilon_t + K\Psi$.

However, because the true distribution is F_t' with mean Υ_t' and standard deviation Ψ', T_t^* is actually far away as much as $\hat{K}$ times of the true standard deviation beyond the true mean value so that $T_t^* = \Upsilon'_t + \hat{K}\Psi'$, where $\hat{K} = (\Upsilon_t - \Upsilon'_t)/\Psi' + K\Psi/\Psi'$. Therefore, the supplier's expected holding and expedition costs with no information sharing incurred at period $t+\tau+1$, C_t^s has the following property.

$$C_t^s = E_{\bar{\epsilon}_t}\left[P\int_{T_t^*}^{\infty}(x - T_t^*)\,dF'_t(x) + H\int_{-\infty}^{T_t^*}(T_t^* - x)\,dF'_t(x)\right]$$
$$= E_{\bar{\epsilon}_t}\left[\Psi'((H+P)L(\hat{K}) + H\hat{K})\right]$$
$$\geq \Psi'((H+P)L(E_{\epsilon_{at,a=1,\ldots,n}}[\hat{K}]) + H\,E_{\epsilon_{at,a=1,\ldots,n}}[\hat{K}].$$

The last inequality is due to the Jensen's inequality for the convex loss function. Because $E_{\epsilon_{at,\,a=1,\ldots,n}}[(\Upsilon_t - \Upsilon_t')] = 0$ and $\Psi \geq \Psi'$, we have $E_{\epsilon_{at,\,a=1,\ldots,n}}[\hat{K}] \geq K$. Since we have $E_{\epsilon_{at,\,a=1,\ldots,n}}[\hat{K}] \geq K$, due to Lemma 2 in Lee et al. (2000), we have $C_t^s \geq C_t^{'s}$. Consequently, we observe that information sharing enables the supplier to reduce its expected inventory holding and expedition costs. Using both the result in equation (4) and that in equation (6), because the buyer decides based on full information and the supplier decides under asymmetric information, the total supply chain cost under no information sharing can be computed as

$$TC_t(l_1, l_2, \ldots, l_n) = \sum_{a=1}^{n} \phi(k)\,(h+p)\frac{\sigma_a}{1-\rho}\sqrt{\sum_{j=1}^{l_a}(1-\rho^j)^2} + E_{\bar{\epsilon}_t}[\Psi'\,[(H+P)L(\hat{K}) + H\hat{K}\,]].$$

Because $C_t^s \geq C_t^{'s}$ and the buyer's cost is invariant to information sharing, we have $TC_t(l_1, l_2, \ldots, l_n) \geq TC_t'(l_1, l_2, \ldots, l_n)$. The minimum value of the piecewise linear function $TC_t(l_1, l_2, \ldots, l_n)$ attained at l_a^* corresponding to one of the extrema of $l_a \in [0, L+1]\ \forall a = 1, \ldots, n$.

By comparing the previous results under full information, we can summarize properties of the optimal structure without explanation about the straightforward proof as follows.

Proposition 2 $\hat{K}=K, \Upsilon_t=\Upsilon_t'$, and $\Psi=\Psi'$ hold when either $l_a=0$ or $l_a=L+1\ \forall a$.

This result indicates that we have $TC_t(l_1, l_2, \ldots, l_n)=TC_t'(l_1, l_2, \ldots, l_n)$ if either $l_a=0$ or $l_a=L+1\ \forall a=1, \ldots, n$. Thus, regardless of whether we adopt the simplified assumption that the supplier does not utilize historical order quantities of a buyer to estimate the actual demand, as in Lee et al. (2000), or not, we have the same the optimal supply chain structure. It is well known that the assumption for simplification tends to exaggerate the benefit of information sharing, as shown in Raghunathan (2001).

3.3 Flexible Production Lead Time

Now, we consider an extended model such that the supplier can reduce its production lead time L by an appropriate investment. As the optimal delivery lead times l_a^* of each buyer a for a given production lead time L should be either one of the extrema of l_ $a\in[0, L+1]$, and the optimal partision, deviding $l_a^*=0$ as G_0 and $l_a^*=L+1$ as G_1, is independent to L, the optimal total supply chain cost function for given $l(L)$, denoting the vector $(l_1^*, \ldots, l_n^*)$, can be expressed as

$$TC(l(L))=\frac{1}{1-\rho}\left\{\phi(k)(h+p)\sum_{a\in G_1}\sigma_a + \phi(K)(H+P)\sqrt{\sum_{a\in G_0}\sigma_a^2}\right\}\sqrt{\sum_{i=1}^{L+1}(1-\rho^i)^2}.$$

Proposition 3 $TC(l(L))$ is monotone increasing in L and ρ, respectively, and is concave in L.

The proposition states that the supply chain performance can improve by reducing the production lead time, and the cost reduces more significantly when the demand at each buyer's site is more positively autocorrelated over periods. The second concavity result will be necessary to find the optimal production lead time in proposition 4. To examine how the optimal production lead time changes to ρ, we can consider a monotone decreasing investment cost function to the production lead time L, given as $\zeta(L)$, within a given boundary condition $\eta\le L\le\xi$, and we have the following result.

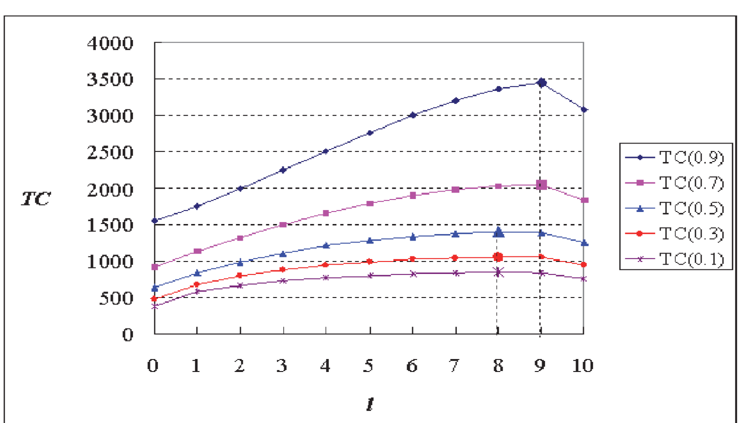
Proposition 4 For given ρ, the optimal production lead time minimizing the total cost $TC(l(L))+\zeta(L)$, denoted by $L(\rho)$ have the following properties as 1) $L(\rho)$ is obtained at one of the extrema of $[\eta, \xi]$ if $\zeta(L)$ is linear and monotone decreasing in L and 2) $L(\rho)$ is decreasing in ρ.

The first property shows that we can easily find the optimal production lead time when $\zeta(L)$ is linear. However, when $\zeta(L)$ is convex and monotone decreasing in L, requiring a heavier investment as the production lead time decreases, the optimal decision $L(\rho)$ is not restricted to one of the two extrema. The second property shows that more investment in reducing the production lead time is needed when demands are more positively correlated as such a property of demand improves the value of the effort to accelerate the production processes of manufacturers.

3.4 Computational Results

To have more insights about the optimal supply chain structure, we have investigated the properties of the performance of the supply chain based on an illustrative example. Cost parameters are set as p = 50, h = 2, P = 25, and H = 1 to represent the lower holding and penalty costs at the supplier's site than the buyers' sites. The production lead time is set as L = 9. The demand process is specified by d = 100, σ = 50, and $0.1 \leq \rho \leq 0.9$. The other parameters are computed as $\Phi(k) = \Phi(K)$, and β = 2.

Figure 2 shows the combined effects of ρ and l on the total supply chain cost, $TC(\rho)$ for given ρ, in the case of a single supplier single buyer model. We can observe its minimum at one of the extrema in $[0, L+1]$. In addition, we can find that switching inventory control policy from RMI to VMI can reduce the supply chain cost more significantly when the costs at the supplier's site are far less expensive than those at the buyers' sites. Interestingly, the system performance may be the worst under the partial location of inventories at different buyers' locations, and the harmful effect of inventory misplacement increases when the demands are more positively correlated over periods, and RMI seems to be close to the worst performance especially for high values of ρ. Considering that the basic cost structure is similar in the model of a single supplier-multiple buyers, by the n times aggregation of the total cost of each

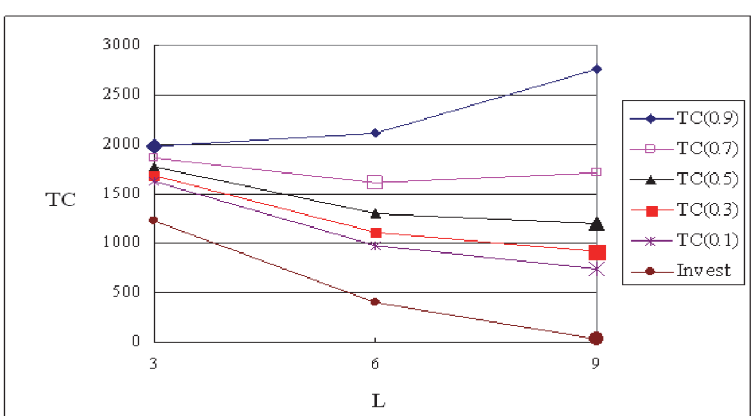


〈Figure 2〉 The Effects of l and ρ on the Total Cost single supplier and single buyer case, we believe that the insight from the computational study in this paper seems to be valid for the case of multiple buyers as well.

Figure 3 shows how both non-linear investment function and different demand correlations over time periods affect the optimal investment decision in the single supplier-single buyer model. TC represents the sum of the investment and the holding and shortage costs. To verify the effect of the investment in the production lead time reduction, we assume that L is in its efficient frontier and the requested investment $\zeta(L)$ follows a quadratic function as $25 * (L - \xi)^2$ for $0 \leq L \leq \xi$, where the current system has a production lead time $\xi = 10$. By marking the optimal investment decision under this convex investment function with larger marks in the figure, we can find that a high level of investment in production time reduction is needed as ρ increases. Such a result coincides with the results in proposition 4 under the linear investment function, except that $L(\rho)$ is not obtained at one of the extrema of $[\eta, \xi]$. Thus, we may recommend that more investment in reducing the production lead time is needed when demands are more positively correlated. In addition, considering the functional form of $TC(l(L))$, the basic cost structure seems to be similar in the model with a single supplier and multiple buyers. Thus, the insight from the computational study in this paper seems to be valid for the case of multiple buyers as well.

Ⅳ. Conclusion

We propose a dyadic supply network model consisting of a manufacturing supplier and a retailer with multiple stores and investigate



〈Figure 3〉 Optimal Choice of L under Different ρ

the optimal supply chain to embrace omnichannel marketing. First, we show that the supply chain works best under a mixture of VMI and RMI, allowing the supplier to monitor all the buyers' inventories and schedule replenishment deliveries under VMI if the supplier's cost is sufficiently lower than the buyers' costs and RMI vice versa. It indicates that the relative magnitude of the buyers' costs to those of the supplier is the critical determinant of the dichotomous feature of the optimal supply chain. Interestingly, our results show that VMI can be a preferable choice even when the supplier's costs are higher than those of buyers, whereas the opposite case is not feasible. From the computational study, we can also find that the partial allocation of inventories between the supplier and the buyer that does not follow the dichotomous feature of the optimal supply chain can degrade the system performance significantly. Thus, when managers do not have clear ideas on the optimal structure and partially allocate inventories among the supply chain participants, the supply chain performance may be the worst. In addition, our results show that the optimal supply chain structure is invariant to the nonstationarity of the demand processes and the asymmetry of information among partners. Thus, we recommend the optimal dichotomous structure of the supply chain as a critical guideline for managers in their omnichannel initiatives regardless of information sharing among different entities in the channel.

The managerial implications of our results for mass merchants like Wal-Mart in the retail industry can be summarized as follows. First, mass merchants under the omnichannel initiative with the advance of digital technologies need to have an aggressive attitude toward the investment in the production process of suppliers for an integrated service experience when demands are more positively correlated. It implies that such merchants may need to support the supplier-driven initiatives to reduce the production lead time as the mass merchant can benefit from such supplier-driven initiatives. Second, our results show that the impact of such an investment in improving the capability of suppliers is highly significant when the mass merchants have many stores in different locations as part of their supply chains. Thus, the benefit from the omnichannel initiatives is more significant for giant retailers like Wal-Mart than others with less market share. Thus, giant retailers will show an aggressive attitude toward omnichannel initiatives. Third, on the effect of the different forms of the investment function of the retailer, including linear and quadratic forms, our results show that the optimal production lead time is not necessarily restricted to one of the two extrema of the feasible region. Thus, mass merchants need to have a flexible attitude toward the investment in supplier-driven initiatives considering their financial conditions and the properties of the effect of IT solutions of SaaS companies.

As the justification of the insights based on the proposed model, we need to point out that it represents the omnichannel dynamics driven by a giant retailer with multiple stores in different locations. The proposed model considers the supply chain structure reported in River Logic (2022) that reflects the industry trend of multiple regional distribution centers relying less on large centralized ones and selecting third-party distributors to supply stores and ship directly to customers. Next, as omnichannel retailers usually charge the same price for the items sold online and offline, regardless of stores, all the retail stores are assumed to have identical cost parameters regardless of the items sold online and offline. In addition, the flexible production lead time reflects the evidence we can observe in Walmart (2019) and WalmartMediaGroup (2020), showing the behavior of mass merchants under omnichannel initiatives requesting manufacturers to provide increased end-to-end visibility in material availability for an integrated service. Thus, our assumptions seem reasonable in representing the omnichannel initiative of mass merchants, and the insight in this paper can be a valuable guideline for retailers with multiple stores which are attempting to embrace omnichannel marketing. However, different models seem to be necessary to address the dynamics of omnichannel initiative from the viewpoint of suppliers. For example, Shao (2021) proposed a different supply chain structure to investigate the impact of competing retailers' omnichannel moves on the supply chain performance from the supplier's perspective. Thus, a careful interpretation of the insight of this paper seems necessary due to the limit of our assumptions.

For further research, it seems possible to consider the different features of the omnichannel specifically as we can witness in the increasingly pervasive activities such as buy-online-and-pick-up-in-store in Gao and Su (2016), the cross-channel return policy in Radhi and Zhang (2019), and online and offline assortment strategy in Shao (2020). We believe it will provide managers more fruitful ideas about how to optimize the supply chain in the presence of a mass merchant driving the omnichannel initiative. In addition, as the emerging trend of omnichannel initiatives includes diverse activities of suppliers such as increasing end-to-end visibility in material availability and satisfying safety requirements, consideration of incentives of different entities in the channel seems necessary. Especially in the initiatives between Nestle and Wal-Mart and between Golden State Foods and Costco, we can observe evidence of why consideration of such activities can provide managers in the retail industry fruitful insight into how they can create value, and we believe it can be a good foundation for extended studies in the future.

References

Baljko, J.(2003), "VMI Study Shows Cost Disparity Among Partners-Inventory Programs Benefit OEMs, Not Suppliers," *Electronic Buyer's News*, 7 April 2003.

Barnes-Schuster D, Y. Bassok, and R. Anupindi (2006), "Optimizing Delivery Lead Time/ Inventory Placement in a Two-Stage Production /Distribution System," *European Journal of Operational Research*, 174(33), pp.1664-1684.

Bell, D. R., S. Gallino, and A. Moreno(2014), "How to Win in an Omnichannel World," *Sloan Management Review*, 56, pp.45-53.

Bourland, K., S. Powell, and D. Pyke(1996), "Exploring Timely Demand Information to Reduce Inventories," *Eur. J. Oper. Res.*, 92, pp.239-253.

Brynjolfsson, Erik, Yu Jeffrey Hu and Mohammad S. Rahman(2013), "Competing in the Age of Omnichannel Retailing," *Sloan Management Review*, 54(4), 2 pp.3-29.

Cachon, G., and M. Fisher(2000), "Supply Chain Inventory Management and the Value of Shared Information," *Management Science*, 46, pp.1032-1048.

Chen, F., A. Federgruen, and Y. S. Zheng(2001), "Coordination Mechanisms for a Distribution System with One Supplier and Multiple Retailers," *Management Science*, 47, pp.693-708.

Fry, M. J., R. Kapuscinski, and T. L. Olsen(2001), "Coordinating Production and Delivery Under a (z, Z)-Type Vendor-Managed Inventory Contract," *Manufacturing & Service Operations Management*, 3(2), pp.151-173.

Gallino, S., A. Moreno, and I. Stamatopoulos(2016), "Channel Integration, Sales Dispersion, and Inventory Management," *Management Science*, 63(9), pp.2813-2831.

Gao, F., and X. Su(2016), "Omnichannel Retail Operations with Buy-Online-and-Pick-up-in-Store," *Management Science*, 63(8), pp. 2478-2492.

Goyal, S. K.(1988), "A Joint Economic-Lot-Size Model for Purchase and Vendor: A Comment," *Decision Sciences*, 19, pp.236-241.

Graves, S.(1996), "A Multi-Echelon Inventory Model with Fixed Replenishment Intervals," *Management Science*, 42, pp.1-18.

Herhausen, D., J. Binder, M. Schoegel, A. Herrm ann(2015), "Integrating Bricks with Clicks: Retailer-Level and Channel-Level Outcomes of Online‐Offline Channel Integration," *Journal of Retailing*, 91(2), pp.309-325.

Jalbar, B. A, J. Gutierrez, and J. Sicilia(2005), "Integer-Ratio Policies for Distribution/ Inventory Systems," *International Journal of Production Economics*, 93-94, pp.407-415.

Lee, H. L, K. So, and C.Tang(2000), "The Value of Information Sharing in a Two-Level Supply Chain," *Management Science*, 46, pp.626-643.

Lee, H. L, and S. Whang(1999), "Decentralized Multi-Echelon Inventory Control Systems: Incentives and Information," *Management Science*, 45, pp.633-640.

Lu, L.(1995), "A One-Vendor Multi-Buyer Integrated

Inventory Model," *European Journal of Operational Research,* 81, pp.312-323.

Radhi, M., and G. Zhang(2019), "Optimal Cross-Channel Return Policy in Dual-Channel Retailing Systems," *International Journal of Production Economics*, 210, pp.184-198

Raghunathan, S.(2001), "Information Sharing in a Supply Chain: A Note on its Value when Demand is Nonstationary," *Management Science,* 47, pp.605-610.

Raghunathan, S., and A. B. Yeh(2001), "Beyond EDI: Impact of Continuous Replenishment Program (CRP) Between a Manufacturer and Its Retailers," *Information Systems Research,* 12(4), pp.406-419.

Raghunathan S.(2003), "Impact of Demand Correlation on the Value of and Incentives for Information Sharing in a Supply Chain," *European Journal of Operational Research*, 146(3), pp.634-649.

River Logic(2022), "10 Things about Omni Channel Supply Chain You may not have Known," https://www.riverlogic.com/blog/10-things-about-omni-channel-supply-chain-you-may-not-have-known (retrieved April 2022).

Schenck, J., and J. McInerney(1998), "Applying Vendor Managed Inventory to the Apparel Industry: What's Working, What's Not and What's Next," *Automatic I. D. News,* 14(6), pp.36-38.

Shao, X. F.(2020), "Online and Offline Assortment Strategy for Vertically Differentiated Products," *IISE Transactions*, 52(6), pp.617-637.

Shao, X.F.(2021), "Omnichannel Retail Move in a Dual-Channel Supply Chain," *European Journal of Operational Research*, 294(3), pp.936-950.

Yao, Y, P. T. Evers, and M. E. Dresner(2007), "Supply Chain Integration in Vendor-Managed Inventory," *Decision Support Systems,* 43, pp.663-674.

Walmart(2019), "Nestle Purina-Beefing up their Walmart Partnership," https://www.walmartmedia.com/nestle-purina-case-study (retrieved Jan 2021).

WalmartMediaGroup(2020), "Inside Look: Walmart media group continues to gain momentum," https://corporate.walmart.com/newsroom/2020/12/16/inside-look-walmart-mediagroup-continues-to-gain-momentum (retrieved Jan 2021).

Zhang, T., L. Liang, Y. Yu, and Y.Yu(2007), "An Integrated Vendor-Managed Inventory Model for a Two-echelon System with Order Cost Reduction," *International Journal of Production Economics*, 109, pp.241-253.

〈Appendix〉

Lemma 1 Suppose that $f(x)$ is concave in x and $x_1 < x_2 < \cdots < x_n$ holds for discrete values of $x_1, x_2, \ldots, x_n$. Then, a piecewise linear function F(x) satisfying $F(x_i) = f(x_i)$ for all $i = 1, \ldots, n$ is concave in x.

Proof To prove the function $F(x)$ is concave we need to show that for any two point a and b the relation $F(\lambda a + (1-\lambda)b) > \lambda F(a) + (1-\lambda)F(b)$ holds for all $\lambda \in (0, 1)$, i.e. the line segment connecting $(a, F(a))$ and $(b, F(b))$ lies below the function $F(x)$. Suppose that, for any given value of $\lambda \in (0, 1)$, $\lambda a + (1-\lambda)b \in (x_i, x_{i+1})$ for some i, and also suppose that an extended line of the line segment $(\lambda a + (1-\lambda)b), F(\lambda a + (1-\lambda)b)$ out of the bound $x \in (a, b)$ intersect with the function $f(x)$ at point $x = a'$ and $x = b'$ such that $a' < a$ and $b' > b$. Then, to prove the concavity of $F(x)$, it suffices to show that the line segment connecting $(x_i, f(x_i))$ and $(x_{i+1}, f(x_{i+1}))$ is placed above the line segment connecting $(a', f(a'))$ and $(b', f(b'))$, where $a' < x_i < x_{i+1} < b'$. Equivalently, it suffices to show that the condition $kf(x_i) + (1-k)f(x_{i+1}) > tf(a') + (1-t)f(b')$ holds for any $0 \leq k \leq 1$, where $kx_i + (1-k)x_{i+1} = ta' + (1-t)b'$.

Let $x_i = \lambda_1 a' + (1-\lambda_1)b'$ and $x_{i+1} = \lambda_2 a' + (1-\lambda_2)b'$ for some for $\lambda_1, \lambda_2 \in (0, 1)$ From the concavity of f we have $f(x_i) > \lambda_1 f(a') + (1-\lambda_1)f(b')$ and $f(x_{i+1}) > \lambda_2 f(a') + (1-\lambda_2)f(b')$ for $\lambda_1, \lambda_2 \in (0, 1)$. By substituting these two conditions to the inequality $kf(x_i) + (1-k)f(x_{i+1}) > tf(a') + (1-t)f(b')$ stated above, we have the following inequalities given as

$$
\begin{aligned}
kf(x_i) + (1-k)f(x_{i+1}) &\geq k(\lambda_i f(a')) + (1-\lambda_1)f(b')) + (1-k)(\lambda_2 f(a')) + (1-\lambda_2)f(b')) \\
&= (k\lambda_1 + (1-k)\lambda_2)f(a') + (k(1-\lambda_1) + (1-k)(1-\lambda_2))f(b').
\end{aligned}
$$

By letting $t = (k\lambda_1 + (1-k)\lambda_2)$, we can find that the condition $kf(x_i) + (1-k)f(x_{i+1}) > tf(a') + (1-t)f(b')$ holds for any $0 \leq k \leq 1$, and the proof is completed.

Lemma 2 1) $\sum_{i=l+1}^{L+1}(1-\rho^i)^2$ is piecewise linear and concave in l, for $l = 0, 1, 2, \ldots, L+1$; 2) $\sqrt{\sum_{j=1}^{l}(1-\rho^j)^2}$ is piecewise linear and concave in l, for $l = 0, 1, 2, \ldots, L+1$.

Proof : 1) For $0 < \rho < 1$, the term $\sum_{i=l+1}^{L+1}(1-\rho^i)^2$ is concave in l, for a continuous variable l because it decreases more sharply as l increases. From lemma 1, a piecewise linear function of it for $l = 0, 1, 2, \ldots, L+1$ is also concave in l.

2) Next, we can easily identify that the term $\sqrt{\Sigma_{j=1}^{l}(1-\rho^{j})^{2}}$ is concave in l for a continuous variable l when $\rho=0$ or $\rho=1$. When $0<\rho<1$, to prove the concavity of the term, it suffices to show that $\sqrt{A(l)+B(l)+Q(l)}-\sqrt{B(l)+Q(l)} \leq \sqrt{B(l)+Q(l)}-\sqrt{Q(l)} \quad \forall l=0,1,2,\ldots,L-1$, where $Q(l)=\Sigma_{j=1}^{l}(1-\rho^{j})^{2}$, $A(l)=1-\rho^{l+2}$ and $B(l)=1-\rho^{l+1}$.

By transferring the terms $\sqrt{B(l)+Q(l)}$ and $\sqrt{Q(l)}$ to the right-hand side and the left-hand side, respectively, and squaring both sides of the condition, we have

$$2\sqrt{(A(l)+B(l)+Q(l))Q(l)} \leq 2Q(l)+3B(l)-A(l).$$

We know $2Q(l)+3B(l)-A(l)>0$ because $Q(l)>0$ and $3B(l)-A(l)\geq 0$ due to the fact that $3B(l)-A(l)=2-\rho^{l+1}(3-\rho)\geq 2-\rho(3-\rho)>0 \quad \forall l=0,1,2,\ldots,L$ for $0<\rho<1$. By squaring both sides and rearranging the result we have $(3B(l)-A(l))^{2}/(8(A(l)-B(l)))-Q(l)\geq 0$, and we can find that this condition is equivalent to $(1/4\rho-1/8)+\Sigma_{i=1}^{l}\left\{\frac{1}{4\rho^{i+1}}+2\rho^{i}-1\right\}\geq 0$.

Because the first term in the parenthesis of the last expression is no less than $1/8$ for $\rho>0$, the condition is satisfied when the second term is nonnegative. Multiplying the second term by a nonnegative term $4\rho^{i+1}$, the condition is reduced to $0\leq 8\rho^{2i+1}-4\rho^{3i+1}-4\rho^{i+1}+1=4\rho(2\rho^{2i}-\rho^{3i}-\rho^{i})+1=1-4\rho(\rho^{i})(\rho^{i}-1)^{2}$. Because the maximum of the term $x(x-1)^{2}$ for given $0<x<1$ corresponds to $4/27$ at $x=1/3$, we have $1-4\rho(\rho^{i})(\rho^{1}-1)^{2}\geq 1-4\rho\left(\frac{1}{27}\right)\geq 1-16/27>0$ and the second square root term is concave in l.

Lemma 3 $\sqrt{f(x)+g(y)}$ is concave in x and y if both $f(x)$ and $g(y)$ are non-negative and concave in x and y, respectively.

Proof: For given (x_1,y_i), (x_2,y_2), and $t'=1-t$, where $0\leq t\leq 1$, we wish to show that $\sqrt{f(tx_1+t'x_2)+g(ty_1+t'y_2)} \geq t\sqrt{f(x_1)+g(y_1)}+t'\sqrt{f(x_2)+g(y_2)}$, which is equivalent to $f(tx_1+t'y_2)\geq t^{2}f(x_1)+t^{2}g(y_1)+t'^{2}f(x_2)+t'^{2}2tt'\sqrt{(f(tx_1)+g(y_1))(f(tx_2)+g(y_2))}$. From the concavity of $f(x)$ and $g(y)$, we have $f(tx_1+t'x_2)+g(ty_1+t'y_2)\geq tf(x_1)+t'f(x_2)+t'g(y_1)+t'g(y_2)$. Therefore, it suffices to show that the term $tf(x_1)+t'f(x_2)+tg(y_1)+t'g(y_2)$ is no less than $t^{2}f(x_1)+t^{2}g(y_1)+t'^{2}f(x_2)+t'^{2}g(y_2)+2tt'\sqrt{(f(tx_1)+g(y_1))(f(x_2)+g(y_2))}$. By eliminating identical terms from both sides of the last inequality condition and squaring both sides, the inequality reduces to $(f(x_1)--f(x_2)+g(y_1)-g(y_2))^{2}\geq 0$. From the nonnegativity of the quadratic form, we have the result.

Proof of Proposition 1: 1) The first term of $TC_t'(l_1, l_2, \ldots, l_n)$, corresponding to the buyers' total cost, is concave in $l_a \in [0, L+1]\ \forall a=1, \ldots, n$ as the concavity is conserved with respect to operations such as addition and multiplication by a positive constant. Next, from the first result in Lemma 2 and the concavity conservation property under multiplication, $\sum_{a=1}^{n}(\sigma_a/(1-\rho))^2$ $\sum_{i=l_a+1}^{L+1}(1-\rho^i)^2$ is concave in $l_a \in [0, L+1]$. Next, the second term of $TC_t'(l_1, l_2, \ldots, l_n)$ is also concave in $l_a \in [0, L+1]\ \forall a=1, \ldots, n$ from the concavity conservation property in Lemma 3. So, $TC_t'(l_1, l_2, \ldots, l_n)$ is concave in l, and its minimum exists at one of the extrema of the feasible region $l_a \in [0, L+1]\ \forall a=1, \ldots, n$.

2) Suppose that each buyer in group B holds its own inventory and the supplier holds inventory for the buyers in group $\overline{B}$. Then, the cost function in part 1) can be written as

$$TC_t' = \sqrt{\frac{\sum_{j=1}^{n}(1-\rho^j)^2}{(1-\rho)^2}}\ \phi(k)(H+P)(\beta\sum_{a\in B}\sigma_a + \sqrt{\sum_{a\in\overline{B}}\sigma_a^2}). \qquad (8)$$

As the cost function is equivalent to proposition 4 of Barnes-Schuster et al. (2006), except that the first term is independent of the other terms of TC_t', we can claim that the system cost is minimized when the supplier holds the inventories for a certain buyer group $\overline{B}$ with standard deviations smaller than those of the other buyers in the group B, similarly as in Barnes-Schuster et al. (2006). It means that out of the $n+1$ possible candidate partitions, the partition consisting of B with $n-m$ buyers and $\overline{B}$ with m buyers is optimal if the resulting system cost is better than that of any other candidate partitions. So, the conditions in parts (i), (ii), and (iii) can be easily derived based on the formula in equation (8).

Proof of Proposition 3: Considering that β_{ij} is invariant to changes in L, the optimal delivery decision is placed at either one of the extrema of the feasible region $l \in [0, L+1]$ regardless of L, $\sqrt{\sum_{i=l_a+1}^{L+1}(1-\rho^i)^2}/(1-\rho)$ is concave in L based on the second result of lemma 3. Therefore, $TC(l(L))$ is concave in L. The monotone increasing property is trivial.

Next, to prove the monotone increasing property of $TC(l(L))$ in ρ, it is enough to show that the term $A(\rho)$ is monotone increasing in ρ, for given $A(\rho) \equiv \sqrt{B(\rho)}/(1-\rho)$ and $B(\rho) \equiv \sum_{i=l}^{L+1}(1-\rho^i)^2$. By differentiation, we have $\partial B(\rho)/\partial\rho = \sum_{i=l}^{L+1} 2(1-\rho^i)(-i\ p^{i-1})$, and $\partial A(\rho)/\partial\rho = ((1-\rho)\partial B(\rho)/\partial\rho + 2B(\rho))/(2(1-\rho)^2\ \sqrt{B(\rho)})$. As the denominator of the last formula is positive, we need to show that the numerator of it is positive. The numerator can be expressed as $\sum_{i=l}^{L+1} 2(1-\rho)\{(1-\rho^i)(-i\rho^{i-1})\} + \sum_{i=l}^{L+1} 2(1-\rho^i)^2$. The last formula can be represented as $2\sum_{i=1}^{L+1}(1-\rho^i)\{-(1-\rho)i\rho^{i-1}(1-\rho^i)\} = 2\sum_{i=1}^{L+1}(1-\rho^i)\{-(1-\rho)i\rho^{i-1}+(1-\rho)(1+\rho+\rho^2+\cdots+\rho^{i-1})\} = 2\sum_{i=1}^{L+1}(1-\rho^i)(1-\rho)\{(1-\rho^{i-1})+(\rho-\rho^{i-1})+\cdots+(\rho^{i-1}-\rho^{i-1})\}$. As the last term in this equation is positive for $0<\rho<1$, $B(\rho)$ is monotone increasing in ρ. Considering that $TC(l(L)) \propto A(\rho)$, we can find that $TC(l(L))$ is monotone increasing in ρ.

Proof of Proposition 4: 1) As $TC(l(L))$ is concave in L from proposition 3 and the investment cost function $\zeta(L)$ is concave in L, the total cost is also concave in L. Therefore, the optimal production lead time is placed at one of the extrema of the feasible region $[\eta, \xi]$.

2) We prove the result by the contradiction. Suppose that $L_1 = L(\rho_1) > L_2 = L(\rho_2)$ for some $\rho_1 > \rho_2$. Because $\zeta(L)$ is strictly decreasing in L, we have $\partial\zeta/\partial L|_{L=L_1} > \partial\zeta/\partial L|_{L=L_2}$. Next, from the definition of $TC(l(L))$ for given ρ_1 or ρ_2, we have $\partial TC(l(L)|\rho_1)/\partial L > \partial TC(l(L)|\rho_2)/\partial L\ \ \forall L$. So, $\partial TC(l(L)\mid\rho_1)/\partial L\mid_{L=L_1} > \partial TC(l(L)\mid\rho_2)/\partial L\mid_{L=L_1}$. Based on the first result in proposition 3, the last term is greater than $\partial TC(l(L)\mid\rho_2)/\partial L\mid_{L=L_2}$. Finally, from the optimality of $L(\rho)$ at ρ_2, we have $0 = \partial TC(l(L)\mid\rho_2)/\partial L\mid_{L=L_2} + \partial\zeta(L)/\partial L\mid_{L=L_2}$. Combining the previous three results, we have $\partial TC(l(L)\mid\rho_1)/\partial L\mid_{L=L_1} + \partial\zeta(L)/\partial L\mid_{L=L_1} > 0$. Because the last expression contradicts the property of $L(\rho_1)$ minimizing $TC(l(L)|\rho_1)+\zeta(L)$, we have the result.

• The author Hag-Soo Kim is a professor at Dept. of Bus. Admin. Gyeongsang National University. He got a Ph.D. from Northwestern University and published papers in leading journals such as Management Science. His current work includes Big Data strategy of supply chains under information asymmetry, strategy of SaaS solutions under supply chain competition, and Platform based competition in digital business.