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An Optimal Composition of Market Samples for New Product Test Marketing*

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The development of new products is crucial for a business to maintain its competitive advantages, but the introduction of new products is risky. To avoid huge losses from launching unsuccessful products, marketers try to test the market potential of a new product in the early stages of the development process. It is not uncommon for a multi-store retail chain to test the potential profitability of a new product in a few selected stores before it introduces the product to the entire chain. Since test marketing would involve huge costs, marketers need to find a better way to select markets for accurate testing while keeping the number of test markets as low as possible. This study proposes a model to develop an optimal sampling network with which one can identify the optimal combination of markets for test marketing. Based on a simple demand model, an approach to find an optimal sampling network that would lead to an efficient estimate of the market size of a new product is developed. It is also demonstrated that the samples based on the proposed procedure produces a much more accurate estimate of the market size for a new product than randomly chosen samples do.

Key words: test market, new product development, optimal sampling

Ⅰ. Introduction

1.1 Background

New product development has been one of the most important research areas in the field of marketing and operations. As well documented by Urban and Hauser (1993), while successful new products bring rewards to the firms that introduce the innovations, the introduction of new products is risky by nature as an unsuccessful launch can incur a huge loss. According to one estimate cited by Kotler and Armstrong (2010), 90 percent of new products fail and each year companies lose $20 billion to $30 billion on failed food products alone. One illustrative example would be the unsuccessful introduction of “new coke” by Coca Cola Company, which resulted in the

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reintroduction of Coke's original formula, re-branded as Coca-Cola Classic. Thus, it is very important to predict the profitability of new products as early as possible in order to minimize the cost of launching unsuccessful products. In fact, it is common for marketers to go through several stages of test marketing of a new product before the product is rolled out nationally.

Such test marketing typically involves several sequential steps: the simulated laboratory test market, the controlled test market, and the standard test market (Churchill and Brown 2004). While the standard test market is done in the most realistic settings and thus can provide the most reliable result than other test marketing procedures, as a product is sold through regular distribution channels with regular marketing supports, the test would be done in a small number of (carefully selected) local market areas that marketers believe to be representative of the national market, mostly due to cost considerations. According to Urban and Hauser (1993), one would need to pay one and one-half million dollars for test marketing of packaged goods in a one city test market even in 1993. In order to generate more accurate forecasts, one might want to test a product at the national level. However, such national testing is very costly, which might reach $20 million in national failure and sometimes meaningless as it would not reduce the risk associated with national rollout.

One possible compromise would be to test a new product in several areas/markets. With such an approach, one can make more reliable forecasts while avoiding the risk of incurring marketing expenses at the national level. One of the most important issues with this approach is to determine which markets/areas should be selected in order to maximize the accuracy of forecasts for the entire market at the national level. This paper tries to provide an answer to the question using a statistical approach. In other words, this study seeks to build a statistical model to create an optimal sampling network for new product testing: which markets to select as test marketing sites.

The prediction of the performance of a new product has attracted many marketing researchers. Bass (1969) provides a powerful yet parsimonious model for new product sales growth for durable products. Since his seminal paper, many studies have been done to enhance the model for the diffusion pattern of a new durable product. Predicting sales performance for new consumer packaged products was initially studied through laboratory measurements. One of the most frequently cited models is ASSESSOR by Silk and Urban (1978). It is a laboratory measurement model in which the market share of a new brand is obtained by multiplying the cumulative trial rate by the purchase share of the trial consumers. The trial rate is measured in a laboratory environment and the repeat purchase rate can be measured with a survey. Shoemaker and Staelin (1976) provide an outline to measure the sampling error associated with such panel survey-based measures. With the availability of scanner-based data, the focus has shifted from laboratory data to standard market data and econometric modeling of such data. For example, Hahn et al. (1994) provide a new product diffusion model for consumer packaged goods with which one can estimate the long-run market share of a new product with aggregate data. While the aforementioned studies provide ways to predict the future performance of a new product, their predictions are limited to the markets in which the data were collected.

One notable exception is the work by Bronnenberg and Sismeiro (2002). Based on spatial statistics (see, for example, Cressie (1993) for extensive and detailed descriptions), they provide a model with which one can make predictions about the sales performance of brands in markets for which no or poor data exist. Intuitively, if two markets are close to each other, then these markets should have similar sales performance if other factors are held constant. While their study does not deal with the new product issue in particular, their model allows researchers to generate reasonable sales forecasts for untested markets in the context of test marketing.

Related to test marketing, Hitsch (2006) develops a Bayesian learning model which predicts the value of reducing demand uncertainty through test marketing and answers whether to launch a new product and then whether the new product to scrap or stay in the market using US ready-to-eat breakfast cereal industry data.

New product development is one of the major areas of attention in operations field. Some of literatures focus on factors of a new product development process and environment to relate them to new product performance. For example, Ettlie and Pavlou (2006) investigate inter-firm partnership during the joint new product development process to affect dynamic capabilities to impact on NPD performance outcome. Jayaram and Malhotra (2010) examine the impact of process and product concurrency on the NPD project performance using large sample data from multiple industries. Bendoly et al. (2012) find effect of both market and supply chain intelligence quality on new product development performance is contingent with dynamic (or stable) market conditions.

Others in operations literature focus on market demand dynamics of new product. Fisher and Raman (1996) enhance demand forecasting of seasonal fashion products by quick response to early sales with Sport Obermeyer sale and forecasting data. Schmidt and Druehl (2005) suggest the depth and the breadth of new product as a driver of a new product diffusion and substitution, whereas Bass model (1969) understands diffusion as a function of the coefficient of innovation and imitation. Debo et al. (2006) consider remanufactured products and new product together and analyze joint life-cycle dynamics with substitution and supply constraints. Druehl and Schmidt (2008) demonstrate how new product might open a new market and ultimately encroach on an existing market. Taaffe et al. (2008) develop profit maximization model for a firm to serve multiple markets in a single season. In their model, firms must decide which markets to serve before they procure the good and how much marketing effort to put each selected market. Chen et al. (2010) investigated fusion product planning decision and developed optimal market offering model. Considering substitution effects and margins, they find small number of products in the market may be optimal.

1.2 Research Issue

Unlike the existing studies in marketing and operations area on the sales prediction of a new product, our study addresses the issue of which markets should be selected for test marketing. Because introducing a new product into the market is risky, marketers try to test the potential profitability of a new product in the early stages of its development process. The most realistic testing environment involves selling the product through regular distribution channels with the usual marketing support. Test marketing achieves a high level of external validity but such gain comes at a high cost (Cao and Zhang 2018). Given the high level of cost associated with standard test marketing, tests would be done in a few selected submarkets. The issue of which specific submarkets should be included in the test is still an open question to marketers. In this context, our study provides a solution to the problem of which submarkets should be sampled in order to maximize the accuracy of the new product test. In other words, the outcome of our study would be an optimal choice of submarkets for standard test marketing of a new product.

The problem of the optimal choice of submarkets would have two components: (i) determining the size of the test sample, i.e., how many submarkets should be tested and (ii) determining the specific choice of the submarkets given the size of the test sample. For example, suppose there are M submarkets in the entire market. The size of test marketing is a choice of a number from 1 to M, $n \in \{1, \ldots, M\}$. And the configuration of test markets is conditional on the size i.e, the choice of best submarkets for a particular test size n. So the problem can be considered a nested optimization problem as follows.

Outer layer: Find the best size $n \in \{1, \dots, M\}$.
Inner layer: Find the best composition of n submarkets among M markets, conditional on the size of sample n.

The entire problem can be tacked in two backward steps. Note that solving the test size problem in the outer layer is straightforward as long as a general approach to solving the sample composition problem in the inner layer is available In the first step, one solves the inner layer problem with the size of the test sample fixed at an arbitrary number. The solution of the first stage would include the optimal composition of submarkets and the expected payoff value associated with the given size of the test sample. In the second step, one solves the outer layer problem by comparing the payoff values of various test sizes in order to determine the optimal test size. For example, if there are M submarkets, one would compare M payoff values as the possible sample size would be $n = 1, 2, \dots, M$. So the procedure in the second stage would be straightforward once a general approach to solving the first-stage problem is found.

Although the outer layer problem can be solved relatively easily, one needs information on the cost associated with the size of test sample. Such cost would vary across industries and also across contexts. More importantly, one should be able to make a tradeoff between the statistical benefit of improving accuracy by increasing sample size and the financial cost of conducting test marketing in an additional submarket. The eventual choice of the sample size is dependent upon the financial cost of carrying test marketing. Lacking the cost information, we cannot solve the size problem completely. Instead, we build a modeling approach that provides the statistical accuracy for an arbitrary size of sample. Therefore, we leave the cost tradeoff as a job of practitioners as we do not have access to the cost information. This is a limitation of our study. Our study focuses on the inner layer problem: to provide a model for determining the optimal composition of submarkets for new product testing for a given test sample size.

Marketers are well aware that designing a standard test market involves deciding the size of test markets and the selection of test markets (Malhotra 2010, p. 269). However, to our surprise, little research has been done on this issue through a formal modeling. Rather, literature is informal in that it briefly states that test markets should be fairly representative or that the more test markets the better (Malhotra 2010). So our study would be among the initial trials to investigates this issue through a formal modeling.

The project contributes to the academic literature by integrating marketing models for new products with spatial statistics models and optimization models. In addition, the project is expected to benefit marketing practitioners in a very direct way. Given the nature of the problem, the project can be directly applied to the problem of a multi-store retail chain that needs to select the stores for a new product test. For a retailer, shelving new products is risky as the opportunity cost of shelf space is high. Because of the opportunity cost of the shelf space, it is not uncommon for manufacturers to pay slotting allowances to the retailer (Sudhir and Rao 2006) for carrying a new product. As such a high level of opportunity costs would also imply a substantial cost of test marketing, retailers would want to minimize the number of test stores while maximizing the accuracy of the test. This study would help retailers find the optimal selection of stores for test marketing. It can also help market research firms such as Nielsen and IRI by providing a way to determine which area/markets should be included in their information networks.

Ⅱ. Model

2.1 Demand Model

We begin with modeling the econometric specification of the demand for existing products. In the demand model, we distinguish between product-specific parameters and market specific parameters. Market specific parameters are common across products but unique in each market. While the identification of product-specific parameters for a product requires data from that product, the information on market-specific parameters obtained from other products can be borrowed in order to predict the demand for a new product. From the demands for existing products, we identify market specific parameters and apply this information to identify the demand for a new product. Our interest lies on which markets to be sampled in order to maximize the accuracy of the estimates of the product specific parameters for a new product.

Let y_{jkt} be log sales of product j at market k at week t. We assume a simple model for the log demand as follows.

$$y_{jkt} = \alpha_j + \beta_{jk} + \varepsilon_{jkt}, \quad j = 1,\ldots,J,\ k = 1,\ldots,M,\ t = 1,\ldots,T \tag{1}$$

In the model, α_j is the market size of product j and β_{jk} is a product and market specific deviation of the mean level sales from the market size level. We assume that such variations are normally distributed centered around a market specific mean level μ_k and are possibly correlated across markets. Specifically, the vector $\beta_{j\bullet} = (\beta_{j1},\cdots,\beta_{jM})$ follows a multivariate normal distribution, $N(\mu, \Sigma)$. We impose an identification restriction $\mu' 1_M = 0$ where 1_M is a vector of ones of length M. This restriction implies $\mu_M = -\mu_1 - \cdots - \mu_{M-1}$. Note that in this specification the sales of a product can be correlated across market through the correlation in $\beta_{j\bullet}$. Existence of such a spatial correlation structure enables us to explore an optimal sampling network. The last term is an independent error term whose variance is product specific, i.e., $\varepsilon_{jkt} \sim \text{iid}\, N(\mathbf{0}, \sigma_j^2)$.

Let $y = (y'_{1\bullet\bullet}, y'_{2\bullet\bullet}, \cdots, y'_{J\bullet\bullet})'$,
$y_{j\bullet\bullet} = (y'_{j\bullet 1}, y'_{j\bullet 2}, \cdots, y'_{j\bullet T})'$, and
$y_{j\bullet t} = (y_{j1t}, y_{j2t}, \cdots, y_{jMt})'$.

Also let $\alpha = (\alpha_1, \cdots, \alpha_J)'$. The stacked vector of the log sales of all products in all markets for all weeks follows a multivariate normal distribution,

$$y \sim N(m, \Omega)$$

where

$$m = \alpha \otimes 1_{MT} + 1_{JT} \otimes \mu$$

and

$$\Omega = 1_T 1'_T \otimes \Sigma \otimes I_J + \text{diag}(\sigma_1^2, \cdots, \sigma_J^2) \otimes I_{MT}.$$

Then, the log likelihood function is given by

$$\log L\left(\alpha, \mu, \Sigma, \{\sigma_j^2\}\right) = -\frac{MJT}{2}\log(2\pi) - \frac{1}{2}\log|\Omega| - \frac{1}{2}(y-m)'\Omega^{-1}(y-m). \quad (2)$$

We use maximum likelihood approach to estimate the model parameters $(\alpha, \mu, \Sigma, \{\sigma_j^2\})$. Evaluation of the log likelihood function requires evaluation of $|\Omega|$ and Ω^{-1}. As the dimension of the variance-covariance matrix is M*J*T, a direct evaluation of the determinant and the inverse of the matrix is computationally costly for typical values of M, J, and T. To tackle the computational issue, we exploit the block diagonal structure of the matrix to evaluate them. Note that the covariance matrix Ω is a block diagonal matrix. That is,

$$\mathbf{\Omega} = \begin{bmatrix} \Omega_1 & & 0 \\ & \ddots & \\ 0 & & \Omega_J \end{bmatrix}$$

where the product specific variance-covariance matrix, Ω_j, is given by

$$\Omega_j = \mathbf{1_T 1'_T} \otimes \Sigma_\beta + (\sigma_j^2 I_T) \otimes I_M.$$

Then we have

$$|\Omega| = \prod_{j=1}^{J} |\Omega_j|$$

and

$$\Omega^{-1} = \begin{bmatrix} \mathbf{\Omega_1^{-1}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{\Omega_J^{-1}} \end{bmatrix}.$$

In order to compute $|\Omega|$ and Ω^{-1}, we utilize the approach suggested by Magnus (1982), which was also the case in Bronnenberg and Mahajan (2001).

Due to the block diagonal structure of the variance matrix, the log likelihood in equation (2) is given by

$$\log L = -\frac{MJT}{2}\log(2\pi) - \frac{1}{2}\sum_j \log|\Omega_j| - \frac{1}{2}\sum_j (y_j - m_j)\Omega_j^{-1}(y_j - m_j). \qquad (3)$$

We provide the detailed expression for the variance covariance matrix in the Appendix. In the regression representation as shown in equation (1), the design matrix can be constructed as follows. First, the design matrix for the market size is given by $X_\alpha = I_J \otimes 1_{MT}$. That is,

$$X_\alpha \alpha = \begin{pmatrix} 1 & & & \\ \vdots & & & \\ 1 & & & \\ & 1 & & \\ & \vdots & & \\ & 1 & & \\ & & \ddots & 1 \\ & & & \vdots \\ & & & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_J \end{pmatrix}.$$

Let J be the design matrix of μ for product j at week t, which is given by an M by M-1 matrix in the following form:

$$J = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ & 1 & & \\ & & \ddots & \\ 0 & 0 & & 1 \\ -1 & -1 & \cdots & -1 \end{bmatrix} = \begin{bmatrix} I_{M-1} \\ -1'_{M-1} \end{bmatrix}.$$

The particular structure of the design matrix results from the identification restriction, $\mu_M = -\mu_1 - \cdots - \mu_{M-1}$. That is,

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ & 1 & & \\ & & \ddots & \\ 0 & 0 & & 1 \\ -1 & -1 & \cdots & -1 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \\ \vdots \\ \\ \mu_{M-1} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \\ \vdots \\ \mu_{M-1} \\ -\mu_1 - \cdots - \mu_{M-1} \end{bmatrix}.$$

Then, the design matrix of μ is given by $X_\mu = 1_{JT} \otimes J$. So the complete design matrix is given by $X = [X_\alpha \quad X_\mu]$ which has J+M-1 columns.

Next, we specify the spatial covariance matrix Σ. In specifying this M by M matrix, we borrow the specification used by Bronnenberg and Sismeiro (2002). In this specification, the spatial covariance between market i and market j or the (i,j) element of Σ_β, is modeled as follows:

$$\Sigma_{\beta(ij)} = \exp(\theta_1/2 + \theta_2 z_i)\exp(\theta_1/2 + \theta_2 z_j) J_0(\theta_3 \delta_{ij}) \qquad (4)$$

where J_0 is the Bessel function of the first kind and order 0 and δ_{ij} is the distance between market i and j. The covariance between markets i and j can be decomposed into the product of their correlation and standard deviations, i.e., cov(i,j)=sd(i)*sd(j)*corr(i,j). In the above specification, the Bessel function specifies the correlation between two markets. The correlation is a function of the distance between market i and j. Other terms in the covariance specification are on the market specific standard deviations. One may expect that markets with large customer bases may have larger fluctuations in sales volume. So the standard deviation would be larger for markets with large market sizes. We model the standard deviation of market j as a function of the size of market j, z_j. Then we have J+3 variance parameters, $\{\sigma_j^2\}_{j=1,\cdots,J}$, Θ_1, Θ_2, and Θ_3. In addition, we have J parameters for $\{\alpha_j\}_{j=1,\cdots,J}$ and M-1 parameters for μ. We use a numerical optimization approach to maximize the log likelihood function. However, only J+3 variance parameters enter the nonlinear search process. This is because the estimates of linear parameters (α and μ) can be obtained from a GLS estimate, $\left(X'\Omega^{-1}X\right)^{-1}X'\Omega^{-1}y$, once Ω is given.

2.2 Demand for the New Product and Sampling Problem

Next, consider the demand for a new product $y_{N+1,kt} = \alpha_{N+1} + \beta_{N+1,k} + \varepsilon_{N+1,kt}$ where $\beta_{N+1,\bullet} \sim N(\mu, \Sigma)$ and $\varepsilon_{N+1,kt} \sim N(0, \sigma_{N+1}^2)$. Since we are able to estimate μ and Σ from the sales data of existing products, we assume μ and Σ are known when it comes to the prediction of the demand for the new product. Suppose a marketer samples from the first n stores out of M stores (n 〈 M) for the test marketing. Then the sales data from the sample are given by $y_{(1)} = (y_1, \cdots, y_n)'$ where y_n is the time series of sales at market n. Using the sampled data, the marketer can estimate α_{N+1} and σ_{N+1}^2. The sampled data follows a multivariate normal distribution, i.e., $y_{(1)} \sim N(\alpha_{N+1} + \mu_{(1)}, \Omega_{(1)})$ where $\Omega_{(1)} = I_T \otimes \sigma_{N+1}^2 I_n + 1_T 1_T' \otimes \Sigma_{(1)}$. The log likelihood is given by

$$\log L = -\frac{nT}{2}\log(2\pi) - \frac{1}{2}\log\left|\Omega_{(1)}\right| - \frac{1}{2}(y_{(1)} - m_{(1)})'\Omega_{(1)}^{-1}(y_{(1)} - m_{(1)})$$

where $m_{(1)} = \alpha_{N+1} 1_{nT} + 1_T \otimes \mu_{(1)}$. Since μ is known, let us use a new notation for the observations, i.e., $y = y_{(1)} - 1_T \otimes \mu_{(1)}$.

MLE for α_{N+1} and σ_{N+1}^2 are obtained from the first order conditions. Our interest lies in estimating the market size, α_{N+1}. Consider a consistent estimator, $\hat{\alpha}_{N+1} = \sum_{k=1}^{n}\sum_{t=1}^{T} y_{kt}/(nT)$. In order to increase the accuracy of the estimation, one needs to minimize the variance of $\hat{\alpha}_{N+1}$. The optimal sampling would involve finding the best combination of stores for sampling to minimize the variance of the estimator. Note that

$$\mathrm{Var}(\hat{\alpha}_{N+1}) = \mathrm{Var}\left[\frac{1}{nT}\mathbf{1}'_{nT}\,y\mathbf{1}_{nT}\right] = \frac{1}{nT}1'_{nT}(\Omega_{(1)})1_{nT}\,.$$

$$\begin{aligned}
1'_{nT}\Omega_{(1)}1_{nT} &= 1'_{nT}(I_T \otimes \sigma^2 I_n + 1_T 1'_T \otimes \Sigma_{(1)})1_{nT} \\
&= \sigma^2 nT + 1'_{nT}(1_T 1'_T \otimes \Sigma_{(1)})1_{nT} \\
&= \sigma^2 nT + \mathrm{tr}\left[1'_{nT}(1_T 1'_T \otimes \Sigma_{(1)})1_{nT}\right]
\end{aligned}$$

Using the fact that $1_{nT} = \mathrm{vec}(1_n 1_T{}')$ and $\mathrm{tr}(AZ'BZC) = (\mathrm{vec}(Z))'(CA \otimes B')\mathrm{vec}(Z))$, we have

$$\begin{aligned}
&= \sigma^2 nT + \mathrm{vec}(1_n 1'_T)(1_T 1'_T \otimes \Sigma_{(1)})(1_n 1_T) \\
&= \sigma^2 nT + \mathrm{tr}\left[1'_T(1_n 1'_T)'\Sigma_{(1)}(1_n 1'_T)1_T\right] \\
&= \sigma^2 nT + \mathrm{tr}\left[1'_T 1_T 1'_n \Sigma_{(1)} 1_n 1'_T 1_T\right] \\
&= \sigma^2 nT + \mathrm{tr}\left[T 1'_n \Sigma_{(1)} 1_n T\right] \\
&= \sigma^2 nT + T^2 1'_n \Sigma_{(1)} 1_n\,.
\end{aligned}$$

Therefore, in order to minimize $\mathrm{Var}(\hat{\alpha}_{N+1})$, we need to choose the set of sampled locations (1) so as to minimize $1'_n\Sigma_{(1)}1_n$. This result also applies when we use GLS to estimate the market size. That is, one needs to minimize $1'_n\Sigma_{(1)}1_n$.

2.3 Search Algorithm

The optimal sampling involves finding the best composition of the sampled markets to maximize the precision of the market size estimate. This is equivalent to minimize the variance of the estimate. The issue of how many markets should be sampled is the issue of sample size. Obviously, increasing sample size would result in more accurate estimate. Determining sample size would involve the assessment of the tradeoff between the cost of sampling and the benefit of information accuracy. Once the economic value of the information accuracy is given, finding the optimal sampling size is straightforward as researchers can easily evaluate the information accuracy for M different levels of sample size. So the computational requirement is linear in M.

This study does not aim to find the optimal sample size. Rather, this study tries to solve a more complicated issue of finding the best combination of markets for sampling when the sample size is fixed at certain value. The optimization issue involves identifying the best set of markets so that $1'_n\Sigma_{(1)}1_n$ is the smallest among all possible set of markets at given value of sample size. When we choose n markets out of M markets, there are ${}_M C_n = n!/\{M!(M-n)!\}$ possible combinations of stores. For example, when M=67 and n=10, there are more than 2.4E+11 combinations. As enumerating all possible combinations is not computationally possible for such cases, one needs to utilize nonlinear integer programming approaches. This type of discrete optimization problem is classified as nonlinear knapsack problem because the objective function is not a linear function of the market composition due to the existence of nonzero covariance. No analytical solution approach has been developed for this type of problems. Typically, one would need to rely on numerical methods such as exchange algorithm, branch-and-bound method, simulated annealing algorithm, or genetic algorithm to solve this type of nonlinear integer programming models. We use the simulated annealing algorithm in our application. Groenigen and Stein (1998) and Zhu and Stein (2006) apply the simulated annealing algorithm in determining sampling plan for spatial statistics. The main reason to use the simulated annealing algorithm is that it can get out of the trap of local optima easily. Moreover, when the size of sample (n) is realistically large, methods such as branch-and-bound is inapplicable. So simulated annealing is a reasonable choice in our setting.

The simulated annealing is an iterative procedure that involves generation of candidate solution, evaluation of candidate solution, and cooling. Note that the input of the optimization process in our case is the indices of markets selected for test marketing, not a numeric vector. Other aspects such as solution evaluation and cooling are the same as the standard simulated annealing process using numeric inputs. So we need to explain the candidate generation process in more detail. Let A be the set of all markets, i.e., $A = \{1,2,\ldots,M\}$. Suppose the current solution is given by S_k at iteration k. S_k contains n indices of markets to be sampled. In order to generate S_{k+1}, we need to first generate candidate for S_{k+1}. Let B_k be the set of unsampled markets in the current solution. That is, $B_k = A-S_k$. We randomly select a market from S_k and also randomly select a market from B_k and replace the selected market from S_k with the one chosen from B_k to generate T_{k+1}, the candidate solution. Then we decide probabilistically whether to accept the candidate following the typical process of the simulated annealing process. Such a probabilistic nature of the acceptance/ rejection decision process may lead to an inferior solution temporarily but eventually allows the algorithm to get out of local optima.[1)]

Ⅲ. Application

3.1 Data

We apply our model to a case of new product introduction in analgesic category. Aleve was introduced to the market in 1994. Before the introduction of the brand, major national brands such as Tylenol, Advil, and Motrin

1) A detailed description on the simulated annealing process used in the study can be obtained from the authors.

were available in the market. A publicly available store level scanner data set obtained from Dominick's Finer Food retail chain in Chicago area includes store level sales of each UPC in 29 consumer packaged goods categories from 1989 to 1997. The data set includes observations from more than 80 stores. Since the data cover the introduction time of the new product and has observations from multiple stores simultaneously, this data set is chosen to apply our model. Although our application is done within one retail chain, the structure of the problem of an optimal sampling network is identical to the case where marketers find the best selection of submarkets for test in the national level. So we treat each store as a market. We estimate the market size of each brand (α), the mean (μ), and the variance matrix (Σ) of the spatial variations using the sales data of the existing brands for 50 weeks before the introduction of the new brand, Aleve. After we obtain the estimate of the model parameters, we find the optimal sampling network for different levels of sample sizes. Although there are more than 80 stores in the data set, we use data from 67 stores because some of the stores do not have complete observations for the same time periods. Moreover, since not all UPCs have complete observations for all stores, we aggregate UPC level data into brand level data. We include 6 major brands in the set of existing products in the analgesic category. Table 1 describes the weekly store level brand sales. Tylenol has the largest sales followed by Dominick's store brand. Aleve's sales volume is slightly higher than Motrin's and it has the largest variance among all brands.

We model the spatial covariance between two stores (or markets) as a product of their store specific standard deviations and the correlation between them. The store specific standard deviation is modeled as a function

〈Table 1〉 Brand Level Sales

Brand		Log(sales)	
		Mean	Standard Deviation
Existing Brands	Motrin	3.87	0.60
	Tylenol	5.74	0.43
	Advil	5.04	0.53
	Bayer	4.21	0.58
	Excedrin	4.67	0.48
	Dominick's	5.13	0.50
New Brand	Aleve	3.91	0.67

of the market size of the store. In order to proxy the market size of a store, we use average weekly customer traffic to the store. We model the spatial correlation between two stores (or markets) as a function of the distance between them. We compute inter-store distances using the latitude and the longitude information of stores. We use Pythagorean formula with converging meridian. That is, the distance between two stores is computed as $\sqrt{(\Delta\phi)^2+\cos(\phi_m)(\Delta\rho)^2}$ where $\Delta\phi$ and $\Delta\rho$ are differences in longitude and latitude measured in radians. We set $\phi_m = 40*\pi/180$ since Chicago area is located near North 40°. Figure 1 is the histogram of inter-store distances in kilometers. For more than 50% of the store-pairs, the distance is less than 30 km. The distance between the farthest pair is around 100km. We normalize the distance variable by dividing the raw distance by the farthest distance when it comes to the estimation of model parameters. So we have $0 \le \delta_{ij} \le 1$.

3.2 Estimation

We use maximum likelihood estimation. The estimation results are presented in Table 2. As expected, the market size estimates turn out to be larger for brands with high sales level. Tylenol and Dominick's store brand are estimated to have large market sizes. Those brands also have larger brand-specific

〈Figure 1〉 Distribution of Inter-store Distance

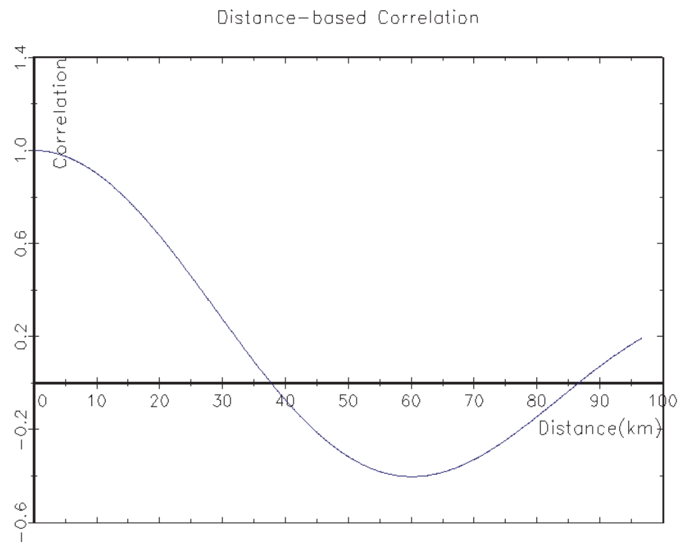
〈Table 2〉 Estimation Results

Brand Specific Parameters				
Brand	Market Size: α_j		Brand Specific Variance: $Log(\sigma_j^2)$	
	Estimate	Standard Error	Estimate	Standard Error
Motrin	3.8929	0.1029	-1.4890	0.0246
Tylenol	5.6658	0.1018	-2.8578	0.0260
Advil	5.0512	0.1020	-2.4130	0.0248
Bayer	4.1786	0.1024	-1.9088	0.0249
Excedrin	4.6167	0.1022	-2.0895	0.0249
Dominick's	5.2006	0.1018	-2.8182	0.0267

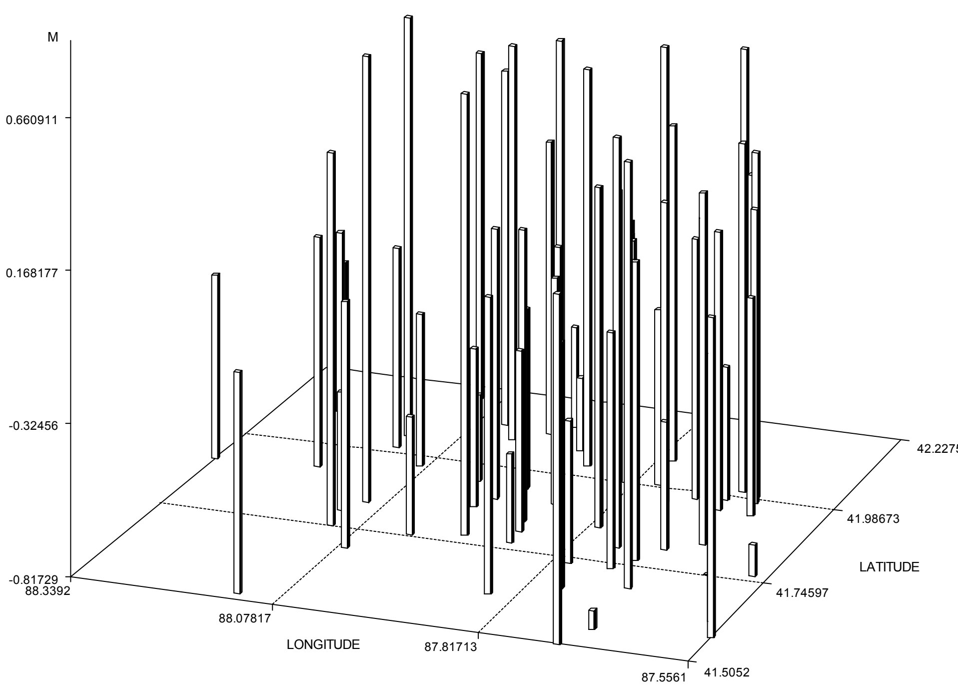
Common Parameters				
Standard Deviation Parameters	Θ_1		Θ_2	
	Estimate	Standard Error	Estimate	Standard Error
	-2.4632	0.4930	0.2318	0.0590
Correlation Parameter	Θ_3			
	Estimate	Standard Error		
	6.1634	0.2672		

variance. The covariance between stores is characterized by three parameters, Θ_1, Θ_2 and Θ_3 as specified in equation (4). The store specific standard deviation is characterized by Θ_1 and Θ_2 and the correlation is characterized by Θ_3. As expected, stores with larger market sizes are estimated to have larger standard deviation as indicated by the positive estimate of Θ_2. Using the estimate of Θ_3, we compute the correlation between stores as a function of distance whose result is plotted in Figure 2. As presented in Figure 2, stores in near distances tend to have a positive correlation in sales. The correlation decreases as the distance increases. As distance becomes larger than 35km, the correlation becomes slightly negative. Finally, as stores are located very far, the correlation becomes close to zero. This pattern of distance-based correlation found in our application is consistent with the result from Bronnenberg and Sismeiro (2002). They also report a similar pattern. Since most of stores are located each other within 40km in our data set as presented in Figure 1, we would observe a positive correlation for dominant number of pairs of stores.

We also have 66 estimates of market specific mean sales levels, μ. Figure 3 plots the estimate of μ_k in market k with its longitude and latitude coordinates. It turns out that



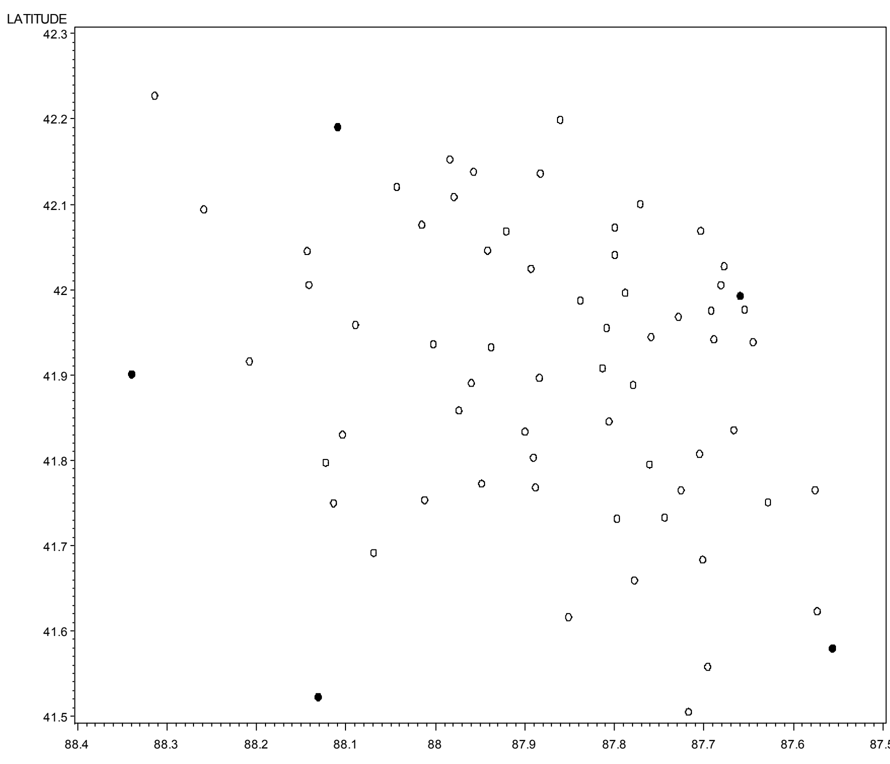
〈Figure 2〉 The Correlation between stores



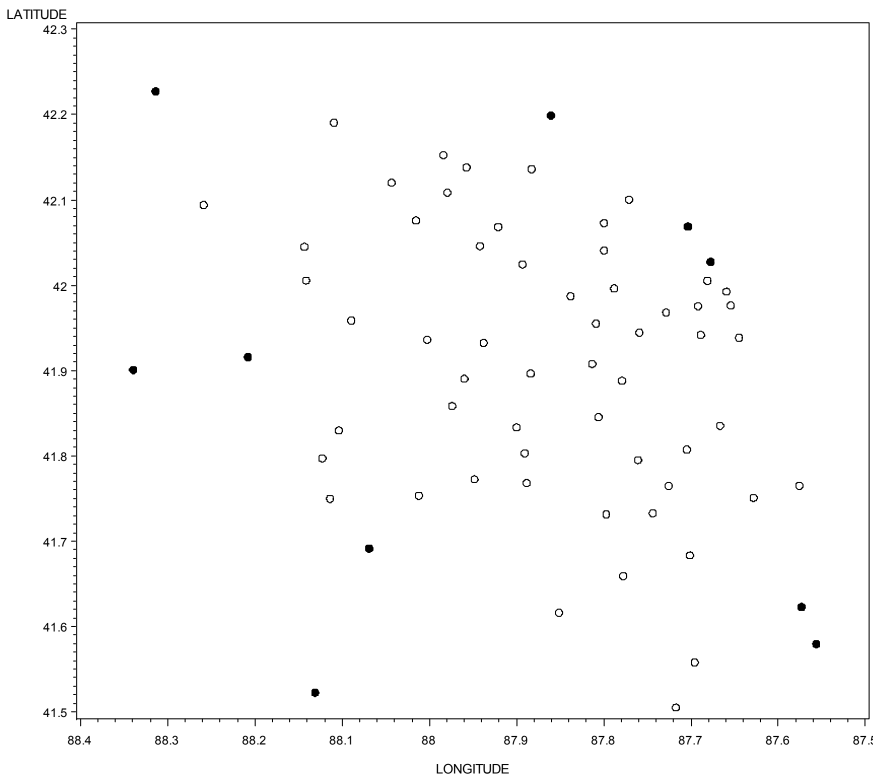
〈Figure 3〉 Estimate of Mean (μ) of Spatial Variation stores located in north area tend to have positive and larger market specific sales. Stores in south side have relatively negative and low sales.

3.3 Optimal Composition of Submarkets

Using the estimate of Σ, we try to find the optimal set of stores for test marketing. We conduct the exercise for two different sample sizes, 5 and 10. When we sample 5 stores out of 67 stores, we end up having stores in the optimal sample as displayed in Figure 4. Each white circle represents the location of a store. Black circles are the locations of the stores that are included in the set of sampled stores. Figure 5 presents the optimal selection outcome when 10 stores are sampled. In our results, sample stores happen to be located outer side of the region where the retailer operates. Such an output might be related to the pattern of sales correlation among stores as shown in Figure 2. Note that the optimal composition of stores are done by minimizing the sum of



〈Figure 4〉 Optimal Sampling Network for n=5



〈Figure 5〉 Optimal Sampling Network for n=10

variance covariance matrix, $1_n' \Sigma_{(1)} 1_n$, The objective function would be smaller when the elements in the variance covariance matrix are small or even negative. The larger the distance between a pair of stores, the smaller the sales correlation between them. So we conjecture that the pattern in Figure 4 and 5 might be related to the correlation pattern in Figure 2.

Next, we demonstrate the benefit of using our model in predicting the market size. Since the sales data of the newly introduced brand, Aleve, are available in Dominick's data, we can assess the accuracy of the prediction of the market size from samples. Using the post-introduction data of weekly sales for 50 weeks from all 67 stores, we find that Aleve's realized market size (α_{N+1}) is 3.9124. Suppose the marketer decides to conduct test marketing for this brand to figure out the market size. Further suppose the test marketing needs to be done in five stores only. When the proposed approach is utilized, the predicted market size using the data from the five stores selected by the proposed model is 3.9171 as presented in Table 3. So the absolute deviation of the prediction from the realized true value is 0.0046.

What if the marketer chooses five stores randomly instead of our approach? We computed the market size estimates from randomly selected samples. We repeat such computation 10 times. As presented in Table 3, the estimate using data from randomly chosen sample stores are much less accurate than that from the proposed approach. The average absolute deviation of the prediction based on random samples from the true value is 0.0814, which is 18 times larger than the error of the proposed approach. Increasing sample size from five to ten does not necessarily leads to a decrease in such gap although the absolute magnitude of the deviation decreases. The absolute deviation of the market size estimate based on the proposed approach is 0.0005. In contrast, the average of the absolute deviations of the market size estimates based on randomly selected stores is 0.0291. Intuitively,

〈Table 3〉 Performance of Sample

Market Size of the New Product: α_{N+1}				
True Value: market size estimate using data from all stores	3.9124			
	Sample Size=5		Sample Size=10	
	Prediction	Absolute Deviation from True Value	Prediction	Absolute Deviation from True Value
Optimal Sampling Network	3.9171	0.0046	3.9129	0.0005
Randomly Chosen Stores	3.9065	0.0059	3.9410	0.0286
	3.9408	0.0283	3.9738	0.0613
	3.9409	0.0285	3.9449	0.0324
	3.8346	0.0779	3.9093	0.0031
	3.7632	0.1493	3.9382	0.0258
	3.8587	0.0537	3.8842	0.0282
	4.1159	0.2035	3.9445	0.0320
	3.9865	0.0740	3.8743	0.0381
	3.8094	0.1031	3.9095	0.0029
	3.8226	0.0898	3.9508	0.0384
	Average	0.0814	Average	0.0291

if the sample size is very large, the magnitude of the absolute deviation would decrease and eventually disappear if the sample size is exactly the same as the number of all stores. However, in realistic situation, test marketing includes only a small number of stores because of the cost consideration. Under such circumstances, the data from the set of stores chosen by the proposed approach would generate a much more accurate result than data from a random set of stores would.

Ⅳ. Conclusion

This study proposes a model for an optimal composition of submarkets for test marketing of a new product. Since test marketing would involve huge costs, marketers need to find a better way to select markets for accurate test while keeping the number of test markets as low as possible. Our approach help marketers identify the optimal combination of markets for test marketing. Specifically, based on a simple demand model, we develop an statistical approach to find an optimal composition of submarkets for test market that would lead to an efficient estimate of the market size of a new product. It is also illustrated that the samples based on the proposed procedure produces a much more accurate estimate of the market size for a new product than randomly chosen samples do.

Note that our proposed model is applied to a data set obtained from a retail chain in Chicago. So one might expect different empirical result when applying the model to a nation wide setting. So the particular patterns obtained in our study as shown in Figure 2 , 4, and 5 should be interpreted in the particular context the model is applied to. However, we expect that the spatial modeling approach used in our study can be well extended, if not directly utilized, to such a national level setting, as done in Bronnenberg and Sismeiro (2002).

To enrich our findings, one possible extension of the model can be done in enriching the specification of the demand model by including the marketing mix variables such as prices. With such extension, marketers can identify the best selection of markets for test marketing not only for precise estimation of the market size of a new product but also for accurate estimation of sensitivity parameters to marketing activities. In such a case, the market- specific heterogeneity in sensitivity to marketing activities could be considered and the model should deal with the issues of how many markets and which markets to sample to predict accurately the entire market heterogeneity. Such model could be a combination of Bronnenberg and Sismeiro (2002) and this study. Another direction for future research could be documenting characteristics of the optimal composition of market samples by describing the particular characteristics of markets included in the samples. Such analysis could be done with a richer specification of spatial covariance structure.

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〈Appendix〉 Derivation of Variance-Covariance Matrix

Consider a matrix $\mathbf{G} = \mathbf{L} \otimes \mathbf{B} + \mathbf{ab}' \otimes \mathbf{A}$ where L is a nonsingular q by q matrix, A and B are square matrices of order p, and a and b are vectors of q. According to Magnus (1982), the determinant and the inverse of that matrix can be computed in the following way.

$$|\mathbf{G}| = |\mathbf{L}|^{p} |\mathbf{B}|^{q-1} |\mathbf{C}|$$

where $C = B + \lambda A$ with $\lambda = b' L^{-1} a$.

$$G^{-1} = L^{-1} \otimes B^{-1} - L^{-1} ab' L^{-1} \otimes E$$

where $E = \begin{cases} C^{-1}AB^{-1} = B^{-1}AC^{-1} = B^{-1}AB^{-1} & ,\text{if } \lambda = 0 \\ \dfrac{1}{\lambda}(B^{-1} - C^{-1}) & ,\text{if } \lambda \neq 0 \end{cases}$

In our case, q = T, p = M, and $\mathbf{G} = \mathbf{\Omega}_j$, $\mathbf{L} = \sigma_j^2 \mathbf{I}_T$, $\mathbf{A} = \mathbf{\Sigma}$, $\mathbf{B} = \mathbf{I}_M$. Then the term λ in Magnus (1982) becomes $\mathbf{1}'_T(\sigma_j^2 I_T)^{-1}\mathbf{1}_T = T/\sigma_j^2$. So, $C = B + \lambda A = I_M + T\dfrac{1}{\sigma_j^2}\Sigma$. This leads to the following expression for the determinant of the product specific variance matrix.

$$|\Omega_j| = |\sigma_j^2 I_T|^{M} |I_M|^{T-1} \left| I_M + T\frac{1}{\sigma_j^2}\Sigma_\beta \right| = (\sigma_j^2)^{MT} \times \left| I_M + T\frac{1}{\sigma_j^2}\Sigma_\beta \right| .$$

And E becomes

$$E = (B^{-1} - C^{-1})/\lambda = \left[I_M - (I_M + T\frac{1}{\sigma_j^2}\Sigma)^{-1} \right] \Big/ (T\frac{1}{\sigma_j^2}) = \frac{\sigma_j^2}{T} \left[I_M - (I_M + T\frac{1}{\sigma_j^2}\Sigma)^{-1} \right] .$$

Therefore the inverse of the product specific variance matrix can be computed from the following expression:

$$\begin{aligned} \Omega_j^{-1} = (\sigma_j^2 I_T)^{-1} \otimes I_M - (\sigma_j^2 I_T)^{-1} 1_T 1'_T (\sigma_j^2 I_T)^{-1} \otimes E &= \frac{1}{\sigma_j^2} I_{MT} - \frac{1}{(\sigma_j^2)} (1_T 1'_T) \frac{1}{\sigma_j^2} \otimes E \\ &= \frac{1}{\sigma_j^2} I_{MT} - \frac{1}{(\sigma_j^2)} (1_T 1'_T) \frac{1}{\sigma_j^2} \otimes E \\ &= \frac{1}{\sigma_j^2} I_{MT} - \frac{1}{\sigma_j^2 T} (1_T 1'_T) \otimes \left[I_M - (I_M + \frac{T}{\sigma_j^2}\Sigma)^{-1} \right] \end{aligned}$$

Due to the block diagonal structure of the variance matrix, the log likelihood in equation (2) becomes

$$\log L = -\frac{MJT}{2}\log(2\pi) - \frac{1}{2}\sum_j \log|\Omega_j| - \frac{1}{2}\sum_j (y_j - m_j)\Omega_j^{-1}(y_j - m_j) .$$

신제품 테스트 마케팅을 위한 최적 시장 표본 구성*

송인성** · 양홍석***

요 약

기업의 경쟁 우위 확보에 있어서 성공적인 신제품 개발이 매우 중요한 역할을 하지만, 신제품 성공 여부의 불확실성에 따른 커다란 위험이 존재한다. 따라서 실패한 신제품에서 비롯되는 커다란 손실을 예방하기 위해 마케터는 신제품 개발의 초기 단계에서부터 해당 제품의 성공 가능성을 테스트하게 된다. 복수의 점포를 운영하는 소매 유통 체인에서도 신제품을 체인 전반에 출시하기 전에 몇몇 선택된 테스트 점포에서 제품을 시험 판매하여 그 제품의 수익성을 사전에 평가하는 것이 일반적이다. 이러한 테스트 마케팅 활동 자체도 상당한 비용을 수반하므로, 테스트 점포의 숫자를 최소한으로 유지하면서도 보다 정확한 정보를 획득할 수 있는 테스트 점포들을 골라낼 필요가 있다. 본 연구는 테스트 마케팅을 위한 최적의 테스트 마켓을 선별하는데 활용되는 최적 표본 네트워크를 찾는 모형을 개발하였는데, 간단한 수요 모형을 기반으로 하여 신제품의 시장 크기에 대한 효율적인 추정을 할 수 있는 최적 마켓 조합을 찾는 접근 방법을 제시하였다. 실제 시장 자료에 적용해 본 결과, 신제품의 시장 크기를 추정함에 있어본 연구에서 제시한 접근 방법이 단순 임의 표본보다 훨씬 더 정확한 예측 결과를 산출하는 것으로 나타났다.

주제어: 테스트 마켓, 신제품 개발, 최적 샘플

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